HELPS TO

GEOMETRICAL DRAWING

PART I

HELPS TO

GEOMETRICAL DRAWING

PART I

LINEAR DRAWING

Plane Geometrical figures, Scales, Conic sections and other plain curves.

Intended for the use of students of the Engineering Colleges and other Technical Schools in India, of the Survey Schools in Bengal, Behar and Orissa and for Art Students.

BY

SURENDRA KUMAR BASU, B.C.E.

Late Professor B. E. College, Sibpore in charge of drawing for over fifteen years. Author of "Helps to Building Construction" "Helps to Geometrical drawing part II (projection)" and "Specification and other useful notes on Building Construction."

Second Edition

S. K. LAHIRI AND CO.,
56, COLLEGE STREET, CALCUTTA.
1924

Price 1/8

Printed and Published by [illegible] K. BANERJEE at the COTTON PRESS, 57, HARRISON ROAD,
CALCUTTA for Messrs. S. K. Lahiri & Co.

PREFACE.

(*Second Edition.*)

The want of a book on Geometrical Drawing suitable for Engineering Colleges and Schools and other Technical Schools in Bengal has long been felt. Problems collected by my predecessors and myself for teaching Geometrical drawing to the Engineer and Overseer students are, therefore, offered in a book from to the students of Bengal.

The course of Geometrical drawing for the Engineer and Overseer students include both the practical plane and solid geometry. The practical plane geometry is suited for school course and is treated in part I. Besides plane geometrical drawings this part contains a chapter on scales, fully dealing with the simple scales, the diagonal scales, the comparative scales and the straight vernier scales.

Some of the problems in this book have been compiled from other books on the subject and some new problems have heen added which is found useful to the students during my long experience in the teaching of geomeraical drawing to the students of the Sibpore Engineering College. The book has been prepared with the object of helping the students to learn the subject without any help from teachers. The survey students and the pleaders intending to appear in the survey examination will find this book useful. Inspite of my best efforts to correct all errors I am sure there are some which have escaped my notice. I shall be grateful to the teachers and other readers of this book if they will kindly point out any mistake or omisssons in the book. I shall also gratefully receive all suggestions for improvement of the future editions.

14th June, 1924.
SIBPORE.

SURENDRA KUMAR BASU.

CONTENTS

CHAPTER I.

CHAPTER II.

CHAPTER III.

CHAPTER IV.

Lines and Angles.

CHAPTER V.

Triangles and Quadrilaterals.

CHAPTER VI.

Polygons.

CHAPTER VII.

To inscribe and circumscribe rectilinial figures.

CHAPTER VIII.

Construction of angles and protractors.

CHAPTER IX.

Circles and circles touching right lines and circles.

CHAPTER X.

Areas and Division of Areas.

CHAPTER XI.

PLAIN SCALES, DIAGONAL SCALES AND COMPARATIVE SCALES

CHAPTER XII.

CONIC SECTIONS, ELLIPSE, PARABOLA AND HYPERBOLA

CHAPTER XIII.

PLAIN CURVES OTHER THAN CONIC SECTIONS.

CHAPTER XIV.

ARCHES.

GEOMETRICAL DRAWING

CHAPTER I.

INTRODUCTION.

Practical Geometry or Geometrical Drawing is very useful to those who deal with practical works. The Engineer or the Architect requires its assistance to solve his knotty problems or to explain his methods. The difficult subject of applied mechanics is made easy by the application of graphic statics and an ordinary draughtsman can find out accurately and in less time the *bending moment* of a beam or the tortional resistance of a shaft. If you can delineate the parts of an object with the proportions on paper you can be sure of the practical execution of the design.

The principal requirement in Practical Geometry is the careful and accurate drawing of figures by means of mathematical instruments.

Mathematical or drawing instruments

The instruments should be the best procurable in the market. "Stanley's" drawing instruments are considered by the draughtsmen to be superior to any other and "Harling's" stand next to them. It is wiser to buy the few most essential ones of good quality than to buy a cheaper box containing all the instruments of inferior quality.

The names and the uses of drawing instruments.

The following instruments will suffice for all ordinary works :—

1. Compasses or Dividers, are used for setting off distances or dividing straight lines. The special kind called the "hair-dividers" have one leg which can be adjusted by means of a spring and screw. These are very useful for dividing straight lines. In using the compasses, they should be held at the top between the fore finger and thumb, with one or more fingers under the hinge to increase or diminish the distance between the points gradually; the steel point should be guided by the finger of the other hand to the required point.

2. **Bow-pencil** :—is a small pair of compasses with one leg constructed to hold a pencil. It is used for drawing circles and arcs.

3. **Bow-ink (or pen)** :—similar instrument to a bow-pencil; it has a ruling pen for one of its legs instead of a pencil. It is used for inking in circles and arcs. Both the bow-pencil and bow-ink should have hinged legs, which will enable the legs to be kept as upright as possible to the paper. It will prevent large holes or uneven lines.

4. **Drawing pen** :—This is used for inking in lines, the thickness of which is regulated by a screw, fixed to the nibs. In using the pen, first dip the nibs or blades in water and then wipe the outside surfaces dry; then with a clean steel nib, or quill, or a slip of paper, take up some drawing ink and insert it between the nibs. The proper thickness of the line is obtained by screwing or unscrewing the blades. A few trial lines should be drawn on a separate piece of paper to see the proper consistency of ink and the thickness of the line. The pen must be held steadily at the same angle to the paper and firmly against the ruler, slightly inclined in the direction of the line to be drawn; both nibs should touch the paper and even pressure to be preserved. The motion of the arm in drawing the pen over the line should be made from the elbow and not from the wrist. By attending to these points an equal thickness of line may be secured and rugged edges avoided. If after sometime it is found that the ink does not run freely from the pen, the defect can be removed by passing a slip of paper or the thin blade of a pen knife between the nibs. If the paper is not clean or greasy by frequent touching, clean sharp lines are impossible. The ink should be wiped out from between the nibs before the pen is put away. The upper nibs in good drawing pens are hinged to the handle for the convenience for cleaning and sharpening at intervals on an oil stone. In inking large drawings two drawing pens are necessary one for fine lines and the other for thicker lines and two drawing pens are always found in a complete drawing instrument box.

5. **Knife key** :—A knife key, with a knife at one end, and two pins at the other for tightening or loosening the joints of compasses or bows is very necessary to keep the first three instruments in good order. It has a file in the middle portion and a small chisel projection to be used as a screw driver.

6. Protractor:—The most general use of Protractor is for setting off on paper any given angle. A variety of scales are also given on both sides of the instrument which make it more useful. It is generally a rectangular piece of ivory or box wood 6 inches long and $1\frac{3}{4}$ inches broad. Round three of its edges the angles are marked and numbered in two rows the outside from 0° to 180° from left to right and the inside similarly in the opposite direction. The method of using it will be given afterwards.

7. A pair of set squares:—Two set squares having angles of 45° and 60° respectively are very useful. These are right angled triangles, used to obtain perpendiculars and also for drawing parallel lines. The angles 15°, 30°, 45°, 60°, 75° and 90° can be drawn by their combined use. Ebonite (black) or talc (white transparent) set squares are preferable over wooden pieces as they are not liable to warp.

In some boxes a pair of hinged parallel rulers is given which can not always be trusted for drawing parallel lines as its useful ness depends on the equal tightness of the two hinges.

The following additional instruments are required for large architectural and machine drawings.

8. 9, 10. Compasses with interchangeable pen and pencil legs with a lengthening rod for drawing large circles.

11, 12, 13. Spring bows (pencil, pen, and points) for drawing very small circles or rivet circles or for marking very small distances.

14. A proportional compasses:—Used to reduce or enlarge a drawing in any given proportion. They consist of two equal and similarly formed parts opening upon a centre which is moveable and forming double pair of compasses. To use the instrument the centre is shifted up or down as required, thereby shortening one set of legs and lengthening the other. The distance to be reduced or enlarged is measured off with one set of legs, and the distance shown by the other pair will be the corresponding length reduced or enlarged in a ratio depending upon the position of the centre pin.

Pencils:—Two degrees of hardness should be used H or H H for drawing in the construction lines and F for drawing in the required figure with firmer lines when it is hot to be inked. The pencils should be sharpened to a moderately fine

point and when used should be gently pressed upon the paper. The lead can be best kept sharp on a piece of fine glass paper.

Drawing Boards and T squares:—For drawings to be tinted with colours a drawing board is necessary which should be strong enough to be used as a table when required and at the same time light to be easily moved, and constructed of such wood as will not warp or expand and contract. For the convenience of drawing on the paper mounted or pasted on the drawing board a T-squre is very useful which is a ruler with a cross piece at one end. It is like the letter T in shape. By keeping the stock or cross piece of the T-square pressed closely against the edge of the drawing board and moving it up or down, lines parallel to each other can be easily drawn on the whole length of the paper.

Indian ink:—Indian ink or Drawing ink is used for inking in a drawing. The ink is prepared by rubbing a cake in a saucer. It should be carefully rubbed, free from grit and not too thick. It should be so worked up as to insure a thoroughly black line which can be found out by a few trial lines on a piece of paper. Liquid Indian ink sold in bottles is very convenient as it is always ready for use. It is now obtained of excellent quality in 1 oz or ½ oz bottles with glass stoppers. The best cakes should be genuine Chinese ink in sticks. The advantage of the drawing ink over common ink is that the former dries quickly, does not corrode the pen and the lines can be washed for colouring without any fear of running.

Besides the above a *pencil eraser* and a *pen knife* are required.

The four important instruments together with an ivory protractor, a pair of ebonite set squares, two pencils, drawing ink, eraser and a good pen knife will cost including a box for instruments about Rs. 45.

GENERAL RULES APPLICABLE TO ALL DRAWINGS.

1. Never draw a single line that is not absolutely necessary. This requires little practice and carefulness.

2. Always rule a line from left to right and slope the pencil slightly towards the direction in which it is moving and inclined away from the front which ensure the point of the pencil always touching the edge of the ruler. Press upon the pencil lightly so that the lines need be only just visible.

3. All lines should be drawn sufficiently long at first, to avoid the necessity for subsequently producing them which is not an easy work for a beginner.

4. Always work from the whole to parts, and not from parts to the whole. This is an important principle in surveying as well as plan drawing and is especially to be observed in the construction of scales.

5. Having determined the extent of a line, always rub out the superfluous length.

6. Avoid using eraser more than is necessary as it tends to injure the surface of the paper. After inking in a drawing, use stale bread in preference to India rubber for cleaning it up as it removes the dirt without removing the fibres of the paper.

7. All angles should be set off, and points determined by means of the largest circles which circumstances will allow to be described.

8. The larger the scale is of the drawing the less liable is the result to error.

9. In determining a point by the intersection of circular arcs or straight lines, these should not intersect at less than 30°.

10. All arcs should be inked in first, as it is easier to join a line to an arc than an arc to a line.

11. Keep all instruments perfectly clean; do not leave ink to dry in the drawing pen.

12. For the convenience of inking arcs of circles, it is advisable to connect the arc with its corresponding centre by enclosing the centre point in a small circle and drawing a dotted line from it to the arc terminated by an arrow-head.

13. Every drawing should have one or more long lines along the length and across the breadth of the paper and nearly on the middle of it. All new lines should be laid off from these guide lines.

CHAPTER II.

PRINTING.

A great deal of practice, care and perseverance is necessary to attain perfection in printing. A good style of printing is essential to the production of a really good engineering or topographical drawing, specially the latter as it abounds with the names of towns, villages, &c. Engineers and superior officers need not spend much time in practising good printing which hould be sought after by the draughtsmen and the subordinate anks.

Generally speaking the Block printing is the best for all kinds of headings, being neat and legible. Fancy letters may occasionally be used in topographical drawings but never in Engineering drawings, the plainer the letters are in a drawing the better.

Block Printing may be either upright or sloping. The proportion of breadth to height ranges from the square form, in which the breadth is equal to the height to the elongated, in which the breadth is one third of the height.

First decide the height of the letters to be used for a heading in a drawing in proportion to the size of it. It depends on the taste and experience of the draughtsman to select a good height for the heading of his drawing which will neither appear too small nor too big. For drawings about 26″ × 20″ size half an inch to three quarters of an inch is the proper height and for drawings on paper 40″ × 27,″ $\frac{7}{8}$th″ to $1\frac{1}{4}$″ may be selected

The rectangular forms of letters look better than the square forms on drawings. A neat and symmetrical appearance is arrived at when the breadth of the majority of the letters is $\frac{4}{5}$th of the height. Divide the height selected into 5 equal spaces. Make the breadths of I = 1, J = 3, F,L = $3\frac{1}{2}$, T,W = 5 spaces, M = 4 or 5 spaces as it is drawn thin or thick and of the remaining letters = 4 spaces. The space between each word may be equal to 4 or 5 spaces. Take care that the terminations of all letters should be always flat and never pointed. In elongated letters divide the height selected into 7 equal spaces

and keep the breadths of the letters the same as stated before. They will then look elongated.

The following hints will be found useful ;—

1. The cross stroke of A should be about $\frac{1}{3}$rd up from the bottom.

2. The upper portion of the letter B is little smaller in height and breadth than the lower portion.

3. In C and G the lower termination is exactly below the upper one.

4. The upper strokes of E and Z should be little shorter than the lower ones.

5. The upper diagonal of K meets the perpendicular stroke two-thirds of the way down. The lower diagonal joins the upper one and is so drawn that it would meet the upright line if produced two-thirds of the way from the bottom.

6. The upper curve of S is little smaller than the lower curve.

Small prints in drawing are better done by Italic printing. Rule three parallel lines to regulate the heights of the small letters and capital. The distance apart of the top space is about $\frac{1}{10}$th of an inch and of the bottom space about $\frac{1}{8}$th of an inch; the height practically depends on the size of the drawing. Inclined parallel lines should then be ruled about $\frac{1}{2}$ inch apart to define the slope of the printing.

The beginner should pencil in each letter before inking in; when he has gained sufficient proficiency in printing the penciling may be dispensed with.

In printing in a drawing the letters should be so placed that the words can be read without having to turn the drawing round.

CHAPTER III.

DEFINITIONS AND TERMS USED IN DRAWING.

A few definitions and terms are given below which will be found useful to beginners :—

1. A vertical line is a right line which points towards the centre of the earth and is in the same direction as a string suspended with a weight attached to it. On paper it is drawn up up-right right in front.

2. A horizontal line is a straight line which forms a tangent to the surface of the earth at the point where you are standing. It is a line drawn at right angles to the vertical line.

3. An oblique line is neither horizontal nor vertical.

4. A quadrilateral figure is also called a trapezium when none of its sides are parallel but may have two of its sides equal. Figs. 1 & 2.

5. A trapezoid is a four sided figure which has only two of its sides parallel to each other. Fig. 3.

6. An oblong is a rectangle. Fig. 4.

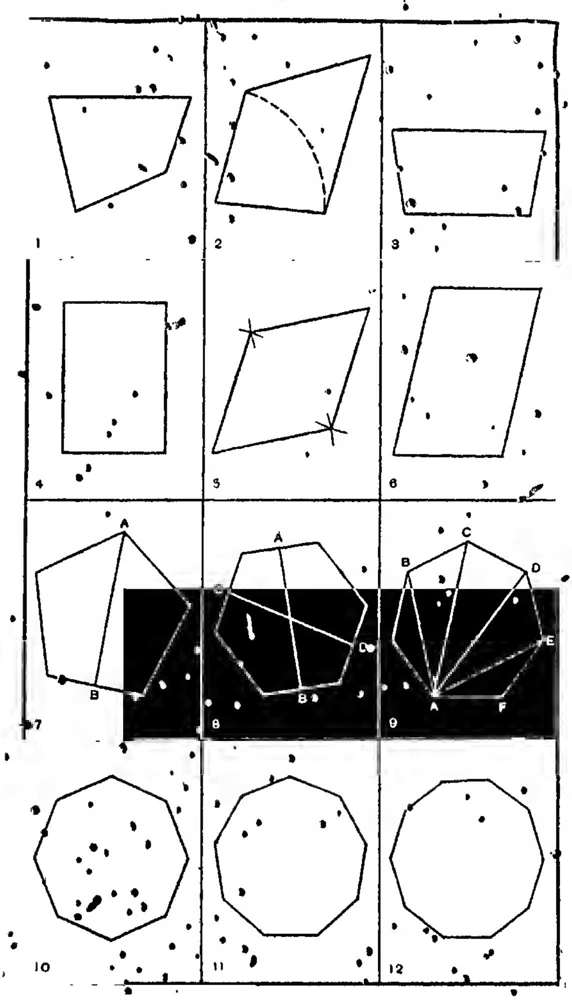
7. A rhombus is a four sided figure which has all its sides equal, but its angles are not right angles, it is a parallelogram whose four sides are equal. Fig. 5.

8. A rhomboid is a four sided figure which has its opposite sides equal but its angles are not right angles. It is a parallelogram. Fig. 6.

9. Polygons are plane figures that contain more than four sides.

Regular Polygons have their sides and angles equal. Eight such polygons are in ordinary use.

1.	Pentagon	... a	five	sided figure.	Fig. 7
2.	Hexagon	... „	six	„ „	„ 8
3.	Heptagon	... „	seven	„ „	„ 9
4.	Octagon	... „	eight	„ „	„ 10
5.	Nonagon	... „	nine	„ „	„ 11
6.	Decagon	... „	ten	„ „	„ 12


1
2
3
4
5
6
A
B
7
A
C
D
B
8
C
B
D
E
A
F
9
10
11
12

7. Undicagon	...	„	eleven	„ „	„ 13
8. Duodecagon	...	„	twelve	„ „	„ 14

Rule to find the angle of a regular polygon :—

To find the angle of a regular polygon divide 360° by the number of sides it contains and subtract the quotient from 180° the remainder is the angle between the sides of the polygon. For instance to find the angle of a nonagon divide 360° by 9 which is 40° and then subtract 40 from 180 which is equal to 140° the angle of a regular nonagon.

10. An ordinate is a line drawn from a point in a curve perpendicular to the axis or the principal diameter as AB in fig. 15.

11. An abscissa is the part of the diameter cut off by an ordinate as BC in Fig. 15.

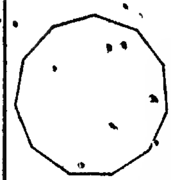
12 A diameter of a polygon is a line which passes through the centre of the polygon and may pass through one corner of the polygon and the middle point of the opposite side when the number of sides of the polygon is uneven, or through the middle points of the two opposite sides of a polygon, when the number of sides of the polygon is even. It bisects the figure, AB in Fig. 7 and AB and CD in Fig. 8.

13. A diagonal of a polygon is a straight line which join, the angular points of the polygon which are not consecutives as AB, AC, AD, or AE. Fig. 9.

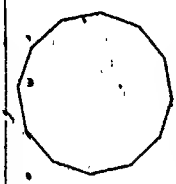
14. An angle is the inclination of two straight lines to each other which meet at a point. This point is called the vertex of the angle. It is described by the revolution of one of the two lines, with the vertex as the centre, starting from the other line which is considered as fixed. Generally the left of the two lines revolves in the direction from the right to the left. The value is measured by degrees on the arc of a circle described of any radius with the vertex as the centre. It is quite a distinct measurement from the linear or space measurement.

The arcs DE and FC show the same angle although FC is longer than DE. Fig. 16.

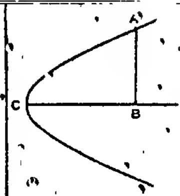
15. When a complete revolution ABEA is divided into 4 equal parts by two straight lines crossing each other at the vertex which is the centre here then each of the quarters is called a right angle, ∠ACB or ∠BCE &c. When the revolution is less than a right angle it is an acute angle as ∠ACG. Fig. 17.



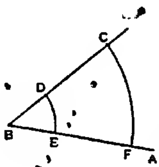
13



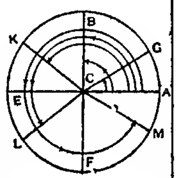
14



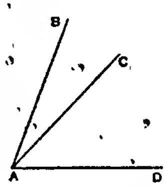
15



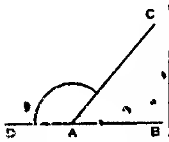
16



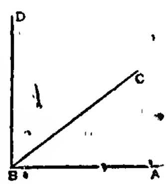
17



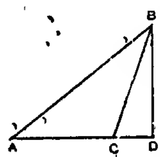
18



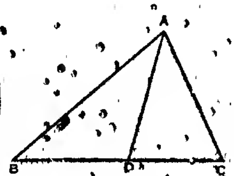
19



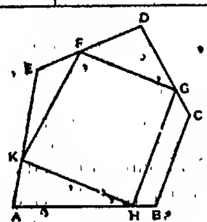
20



20(A)



21



22

When it is more than a right angle but less than two right angles it is an obtuse angle as $\angle$ACK and when it is more than two right angles and less than one revolution it is a reflex angle as $\angle$ACL or $\angle$ACM. Fig. 17.

16. Adjacent angles have a common vertex and one common side as $\angle$BAC and $\angle$CAD. A the common vertex and AC the common side. Fig. 18.

17. The supplement of an angle is the difference between it and the two right angles. $\angle$CAD is the supplement of $\angle$CAB and vicé versá. Fig. 19.

18. The complement of an angle is the difference between it and a right angle. $\angle$CBD is the complement of $\angle$ABC and vicé versá. Fig. 20.

19. Rectilineal figures are those which are bounded by straight lines as triangles, quadrilaterals and polygons.

20. The sum of the sides of any rectilineal figure or the length of the boundary of any figure is called its perimeter.

21. Any side of a triangle, quadrilateral or polygon may be called its *base*. Usually the side which is horizontal or nearly so is taken as the base.

22. The height of a figure is called its altitude. The perpendicular from the opposite angle to the base of a triangle is its altitude, as BD perpendicular to base AC of the triangle ABC, Fig. 20A.

23. The line which divides a triangle into two equal parts from a corner is called the median of the triangle. It is a line from a corner to the middle point of the opposite side as AD, Fig. 21.

24. When a rectilineal figure is drawn inside another rectilineal figure so that all its angular points lie on the sides of the outer figure the former is said to be inscribed in the other, FGHK is inscribed in ABCDE. Fig. 22.

25. When a rectilineal figure is so drawn that its sides pass through all the angular points of another rectilineal figure the outer figure is said to be circumscribed about the inner figure. ABCDE is circumscribed about FGHK. Fig. 22.

The term inscribe is also used as opposed to describe.

CHAPTER IV.

RULES FOR INKING IN DRAWINGS.

1. Given lines to be thin continuous lines.
2. Resulting lines to be thick continuous lines.
3. All construction lines to be thin dotted lines.

LINES AND ANGLES.

1. To bisect a given straight line. Fig. 23.

Let AB be the given straight line. With A as centre and radius more than half the line draw arcs above and below the line. With B as centre and with the same radius intersect the arcs already drawn at C and D. Join CD by a straight line which will not only bisect the line but will be at right angles to it.

2 To draw a perpendicular to a given straight line (a) from a point in the line (b) from a point without it. Figs 24 and 25.

(*a*) Let AB be the given straight line. C is a point near one end of it Take any point O on the upper side of AB. With O as centre and OC as radius describe a circle cutting AB in D. Join D O and produce it to meet the circle at E. Join EC. Then ECD is a semicircle and the angle ECD in the semicircle is a right angle. Fig. 24.

(*b*) Let FG be the given straight line and K the given point outside it. With K as centre and radius more than the distance of it from FG draw the arc LMN cutting F G in L and N. With L and N as centres and with radius more than half the distance NL intersect arcs in P. Join KP. Then KP will be perpendicular to FG. Fig. 25.

3. Divide a given straight line into any number of equal parts, say 8 parts. Fig. 26.

Let AB be the given straight line to be divided ; draw a right line AC making any angle with AB. Set off on AC any convenient length 8 times as A1, 12, 23, 34, 45, 56, 67 and 7 C. Join C and B. Draw from the points 7, 6, 5, 4, 3, 2, and 1, lines parallel to CB then AB will be divided into 8 equal parts.

4. Divide a straight line AB, 1' inches long into parts which shall have the ratio to each other of 2 : 3 : 5. Fig. 27.

Draw a straight line AB and measure on it a length of $1\frac{1}{2}$ inches either from a foot rule or from the inch scale on the

back of protractor. Draw AC making any angle with AB. Set off a length 2 + 3 + 5 = 10 times on AC and mark the 2nd, 5th and the last point 10. Join the last point 10 with B. Draw through the 5th and the 2nd points lines parallel to 2 : 3 : 5 in the points D and E. AD : DE : EB :: 2 : 3 : 5. Fig. 27.

5. To find the third proportional between two given lines A and B; *i.e.* find a length C so that A:B:: B:C. Fig 28.

Let A and B be the two given lines. Take any straight line as DE and set off distances from one end of it DF and FE equal to A and B respectively Draw a straight line DG equal to B and making any angle with DE. Produce DG and join GF. From the point E draw EH, parallel to FG meeting DG produced in H. Then GH will be the third proportional between A and B and will be equal to C.

6. To find the fourth proportional between three given lines A, B and C. *i.e.* find a length D, so that A : B::C:D Fig. 29.

Take EF and FG in the same straight line and equal to A and B respectively. Draw a line EH. at any angle with EG and equal to C. Produce EH and join FH. Draw GK parallel to FH meeting EH produced in K. Then HK is he fourth proportional and is equal to D.

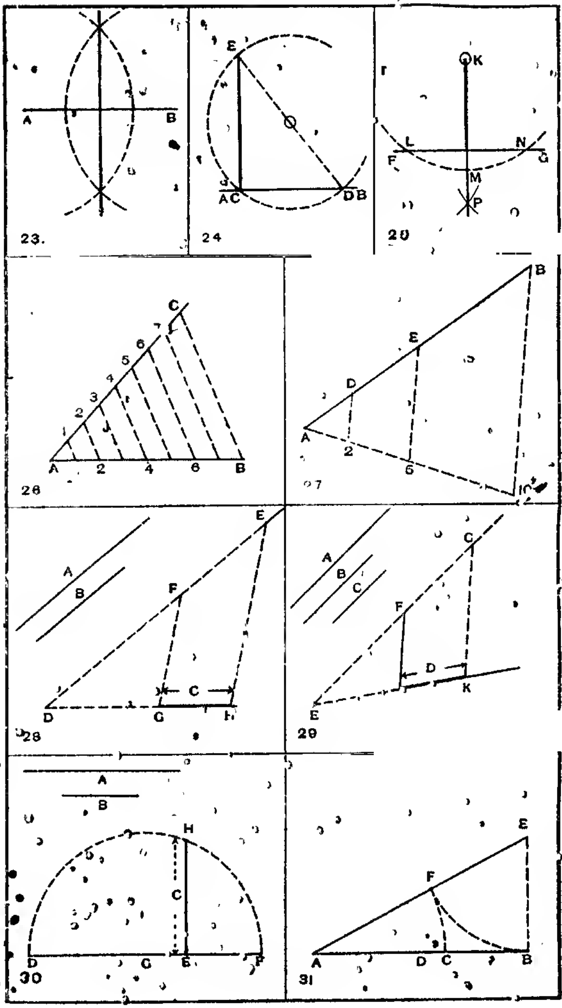
7. To find the mean proportional between two given lines A and B that is to find C so that A:C::C:B. Fig. 30.

Take DE, EF on the same straight line and equal to A and B respectively. On the whole line DF draw a semicircle DHF. From E draw EH perpendicular to DF meeting the semicircle at H. Then EH is the mean proportional between DE and EF *i. e.* between A and B *i. e.* EH is equal to C.

8. Divide a given straight line AB in the extreme and mean ratio that is to find a point C in AB such that AB: AC::AC:CB. Fig. 31.

Bisect AB at D and at B draw BE perpendicular to AB and equal to BD *i. e* half of A B. Join EA. With E as centre and EB as radius draw an arc BF meeting EA in F. With A as centre and AF as radius draw an arc FC meeting AB in C. Then AB is divided in C such that AB:AC::AC:CB.

9. To find a point M in a given straight line AB which shall be equidistant from two given points P and Q without the line. Fig. 32.


A
B
23.
E
O
AC
DB
24
K
L
N
F
G
M
P
25
C
7
6
5
4
3
2
1
A
2
4
6
B
26
B
E
D
A
2
6
10
27
E
A
B
F
C
D
G
H
28
A
B
C
C
F
D
K
E
29
A
B
H
C
D
G
E
F
30
E
F
A
D
C
B
31

Join PQ. Bisect PQ at C. From C draw C M perpendicular to PQ meeting AB in M. Then M is the required point, as MP and MQ are equal.

10. From two given points without a straight line to draw two straight lines to meet the given line and make equal angles with it.

Let P and Q be the two given points and AB the given straight line. Draw PD perpendicular to AB and produce it to E and make DE = PD. Join EQ cutting AB in C. Join PC. Then ∠PCA = ∠QCB that is PC and QC are equally inclined to AB.

11. To bisect a given angle. Fig. 34.

1st method:—Let BAC be the given angle. With A as centre and with any convenient length as radius draw the arc EF meeting AB and AC in E and F respectively. With E and F as centres with any radius draw arcs intersecting in G. Join AG. Then AG bisects the ∠BAC.

2nd method: Fig. 35.—Take any two points D and E in AB; with A as centre and radii AD and AE draw arcs meeting AC in F and G respectively. Join DG and EF which intersec in H. Join AH. Then AH bisects the angle BAC.

12. To trisect a given right angle. Fig. 36.

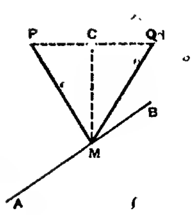
Let ABC be a right angle. With A as centre and with any radius AB draw the quadrant BC. With B and C as centre and with the same radius AB intersect the arc BC in D and E Join AD and AE; then AD and AE will trisect the quadrant.

13. To draw a straight line parallel to a given straigh line and at a given distance. Fig. 37.

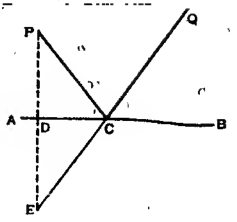
Let AB be the given line and D the given distance tak any two points E and F in the line AB and draw perpendicular EG and FH from these points to the line AB. Make the pe pendiculars equal to D by compasses or by arcs drawn fror the points E and F with radius equal to D. The line passin through G and H is parallel to AB.

N.B. Practically perpendiculars and parallels are drawn b set squares and not by geometrical problems. It requires littl practice to learn the use of set squares one of which is to b held firm to the paper when the other is moved. Fig. 38.

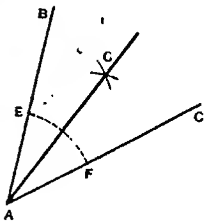
14. To draw a line through a given point parall to a given line.



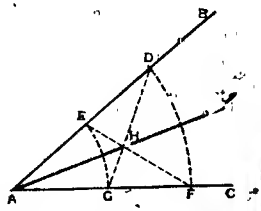
32.



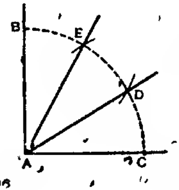
33



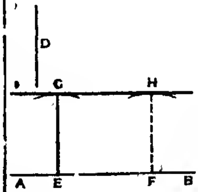
34.



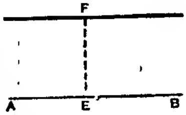
35



36



37.



38

Let AB be the given line and C the given point. From C draw CD perpendicular to AB. Take a point E in AB as far away from the point D as convenient and draw EF perpendicular to AB. Make EF equal to CD. Join CF and produce it both ways. The line CF is parallel to AB and drawn through the given point C. Fig. 39.

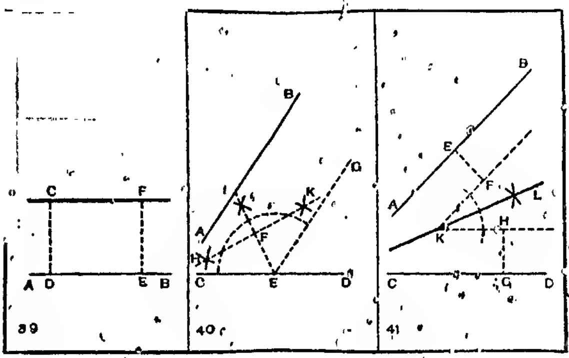
The parallel through C can be drawn as well by a pair of set squares.

15. To determine the direction of the line which would bisect the angle between two converging lines, intersecting beyond the limits of the paper. Figs. 40 & 41.

Let AB and CD be the two converging lines.

1st Method :—On CD take any point E. Draw EG parallel to AB. Bisect the angle CEG by EFL meeting AB in L. Bisect the line EL by HK which intersect it at F. This bisector HK will bisect the angle between the two lines AB and CD when sufficiently produced. Fig. 40

2nd Method :—Take any point E in AB, draw EF perpendicular to AB and of any convenient length. Take another point G in CD ; draw GH perpendicular to CD and equal to EF. Through F and H draw straight lines KF and KH parallel to AB and CD respectively intersecting at K. Bisect the angle FKH by KL. Then KL produced will bisect the angle between AB and CD. Fig. 41.



16. Determine the direction of the line drawn through a given point between two converging lines which would pass through the point where the two lines meet. Fig. 42.

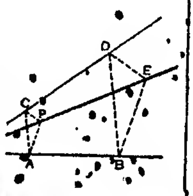
Let AB and CD be the two converging lines and P any point between them. Draw any straight line DB intersecting the two given lines at D and B on one side of the point P and another straight line CA on the other side of the point and parallel to DB. Join CP and AP. Draw DE and BE parallel to CP and AP respectively meeting at E. Join PE which produced will pass through the point where the two convergent lines meet.

17. Through a point P draw a line making equal angles with two converging lines AB and CD. Fig. 43.

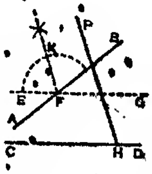
Let AB and CD be the two converging lines Through any point F in AB draw EFG parallel to CD. Bisect the angle EFB by the line FK. Through P draw PH parallel to KF. The line PH makes equal angles with AB and CD.

18. To draw an angle equal to a given angle. Fig. 44.

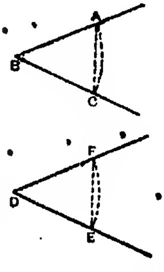
Let ABC be the given angle. Draw a straight line DE. At the point D on the straight line DE an angle is to be constructed equal to ABC. With B as centre and with any radius BA draw an arc AC meeting BC in C. With the same radius and from D as centre draw an arc EF. Set off the chord length AC from the point E on the arc EF as chord EF. Join DF and produce. Then the angle EDF is equal to the $\angle$ ABC.



42



43



44

19. To divide a right angle into 5 equal parts. Fig. 45.

Let ABC be a right angle. Divide one side BC in the extreme and mean ratio in D so that BC : BD : : BD : DC. With B as centre and BD the greater segment as radius draw the quadrant DF cutting BA in F. With C as centre and CB as radius describe the arc BE intersecting the arc DF in E. Then the arc FE is a fifth of the quadrant FD. Set off on FD arcs equal to FE which will divide it into 5 equal parts. The greater segment of the side BC should be towards the right angle.

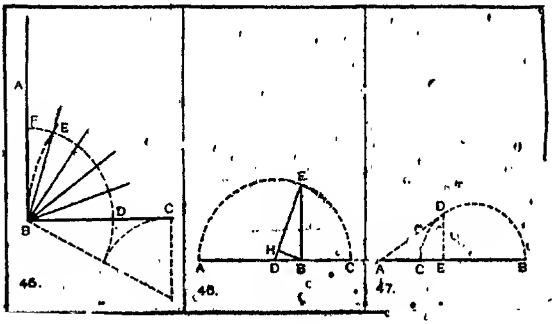
20. To find the Arithmetic, the Geometric and the Harmonic mean between two given lines AB and BC. Fig. 46.

Place the two lines AB and BC as one straight line ABC. Bisect AC in D With D as centre, and radius DA draw the semi-circle AEC. From B draw BE perpendicular to AC meeting the semi-circle in E. Join DE. From B draw BH perpendicular to DE. Then AD is the Arithmetic, BE the Geometric and EH the Harmonic mean between AB and BC.

21. To divide a given straight line harmonically. Fig. 47.

AB is the given straight line. It is required to divide it in two points C and E such that BA : AC : : BE : EC or to find a length AE such that BA—AE : AE—AC : : BA : AC.

Take C any point in AB and on CB describe a semicircle. Draw AD a tangent to the semicircle touching it at D. From D draw DE perpendicular to AB. Then BA : AC : : BE : EC.



22. A given length is $1\frac{1}{2}$ inches long; find lines representing $\sqrt{2}$, $\sqrt{3}$, $\sqrt{6}$ &c. Fig. 48.

Let AB be the given length representing $1\frac{1}{2}$ inches from a scale.

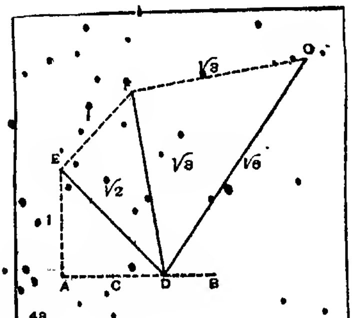
To find the unit length, divide AB into three equal parts in C and D. Then each part represents $\frac{1}{2}$ inch and $AD = 2 \times \frac{1}{2}'' = 1'' =$ unit length. Draw AE perpendicular to AB and equal to AD. Join DE, then DE represents $\sqrt{2}$. At E on ED draw EF perpendicular to it and equal to AB. Join ED; then FD is $= \sqrt{3}$. Draw FG perpendicular to FD at F and equal to it. Join GD. Then GD represents $\sqrt{6}$.

To construct certain angles without the aid of a protractor.

Draw a circle of any radius; divide the circumference into 4 equal parts by two diameters crossing each other at right angles. Then each division is 90°. Set off the radius length on the circumference and join these points with the centre, then each angle at the centre is 60°.

By the use of set squares 45° and 60° the following angles can be easily drawn.

15°, $22\frac{1}{2}$°, 30°, 45°, 60°, $67\frac{1}{2}$°, 75°, 90°, &c.



48

CHAPTER V.

TRIANGLES AND QUADRILATERALS.

23. On a given straight line to construct a triangle similar or equiangular to a given triangle. Fig. 49,

Let ABC be the given triangle and HK the given line, At H on HK draw the angle OHK = ∠ BAC (prop. 18—Chap. IV) and at K draw the angle OKH = ∠ BCA, then the ∠ HOK is equal to the remaining angle ABC and the triangle OHK is equiangular to the triangle ABC.

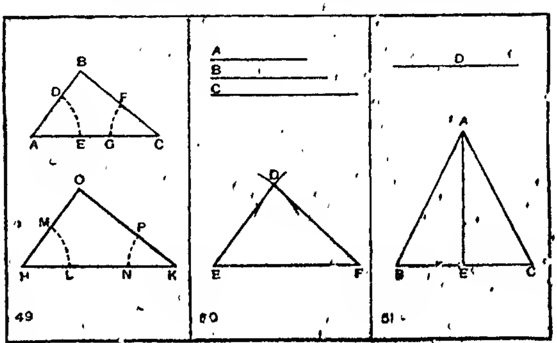
24. To construct a triangle, the three sides being given. Let A, B and C be the three given sides. Fig. 50.

Draw a straight line EF equal to C. With E as centre and radius equal to A draw an arc and with F as centre and radius equal to B draw an arc to intersect the first arc at D. Join DE and DF. Then DEF is the triangle whose three sides are equal to A, B and C.

This problem is useful for plotting the triangles of a survey.

25. To construct an isosceles triangle, the base and altitude being given. Fig. 51.

Let BC be the base and D the given altitude. Bisect BC at E and draw EA perpendicular to it and equal to D. Join AB and AC. Then ABC is the required isosceles triangle.



26. To construct a triangle with two sides equal to two given lines and the included angle equal to a given angle C. Fig. 52.

Let A and B be the two given lines and C the given angle. Draw an angle DEF equal to C, the given angle. Mark off EF equal to B and ED equal to A. Join DF. Then DEF is the required triangle.

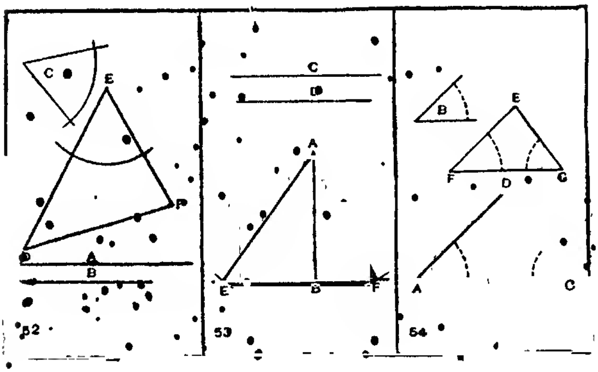
27. To construct a triangle given two sides and an altitude. Fig. 53.

Let AB be the given altitude, C and D the two given sides. Through B one end of the altitude draw EF at right angles to AB. With A as centre and with radii equal to C and D respectively draw arcs cutting EF in E and F. Join AE and AF. Then AEF is the triangle.

28. To construct a triangle, given the vertical angle, one of the base angles and the base. Fig. 54.

Let ∠E be the vertical angle, ∠B, one of the base angles and AC the base.

At A, on AC make the angle CAD equal to ∠B. In E the vertical angle make the angle EFG equal to ∠B. At C draw the angle ACD equal to the angle EGF. Then, the triangle DAC is the required triangle.



29. To construct a triangle, given base, one angle at the base and the difference of the two sides. Fig. 55.

Let AB be the base, $\angle$C the given angle of the base and D the difference of the two sides.

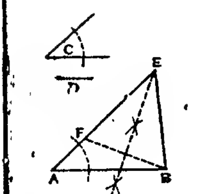
At A on the line AB make an angle equal to the given angle C. Mark off AF equal to D, the given difference of the two sides. Join FB. Bisect FB at right angles by a line meeting AF produced in E. Join EB. Then AEB is the required triangle.

30. To construct a triangle given the base, the vertical angle and the sum of the two sides. Fig. 56.

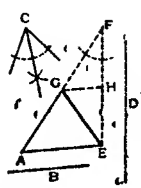
Let B be the base, C the vertical angle and D the sum of the two sides or perimeter minus the base. Draw any line AF and make it equal to D. Bisect the given vertical angle C. At F on the line FA construct an angle AFE equal to half the vertical angle C. From the point A and with radius equal to B draw an arc cutting FE in E. Join AE. Bisect FE at right angles by GH meeting AF in G. Join GE. Then GEA is the required triangle.

31. To construct a triangle the base, altitude and the vertical angle being given. Fig. 57.

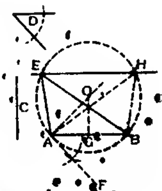
Let AB be the base, C the altitude and $\angle$D the vertical angle. At A on AB make the angle BAF equal $\angle$D. At A draw AO perpendicular to AF. Bisect AB at G and draw GO perpendicular to AB meeting AO in O. Then with centre O



55.



56



57

and radius OA draw a circle AEHB. The angle in the segment AEHB is equal to ∠ BAF = ∠ D. Draw EH parallel to AB at a distance C from it and cutting the circumference in E and H. Join AE, BE or AH and BH. Then AEB and AHB are the two triangles with the given conditions.

32 Draw a right angled triangle, given the hypotenuse and one of the sides. Fig. 58

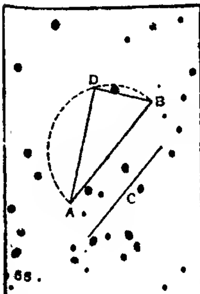
Let AB be the hypotenuse and C the length of one side. Describe a semicircle ADB on AB. With A as centre and C as radius intersect the arc of the semicircle in D. Join AD and DB. Then ADB is the required triangle.

33. Draw a right angled triangle, given the hypotenuse and the perpendicular let fall on to it from the opposite angle. Fig. 59.

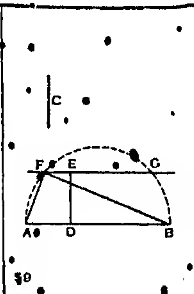
Let AB be the hypotenuse. Draw a semicircle on AB. Let C be the given altitude. Draw FG parallel to AB at the distance C from it (prob. 13—Chap. IV). FG cuts the circumference at F and G. Join AF, FB or AG, GB. Then AFB or AGB is the triangle with the given conditions.

34. Construct a triangle given two sides and the included median. Fig. 60.

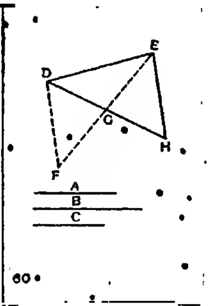
Let A and B be the two sides and C the included median Draw a triangle DEF with the side DF = A, DE = B and EE = 2C, *i e.*, double of the given median. Bisect the side FE in G. Join DG and produce it to H making GH equal to DG. Join EH. Then DEH is the required triangle.



58



59



60

35. To construct a triangle the base and the ratio of the angles being given. Fig. 61.

Let AB be the base and the ratio of the angles be as 2 : 3 : 4.

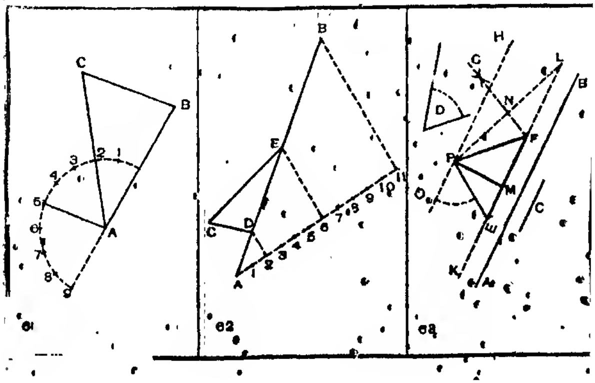
Produce BA; with A as centre and with any radius describe a semicircle. Divide this semicircle into $2+3+4=9$ equal parts (first into 3 equal parts by setting off the radius length on the arc and then divide each part into 3 equal parts by trial) mark the divisions as 1, 2, 3, 4, 5, &c., Join A2 and produce. Join A5. From B draw BC parallel to A5 meeting A2 produced in C. Then ACB is the required triangle.

36. Construct a triangle, the perimeter and the proportion of the sides being given. Fig. 62.

Let AB be the perimeter and the ratio of the sides be as 2 : 4 : 5. Draw a line at an angle to AB and set off any convenient length $2+4+5=11$ times on it. Join B and the 11th point. From the 6th and the 2nd division draw 6E and 2D parallel to B11 cutting AB in E and D. With D and E as centres and with DA and EB respectively as radii draw arcs cutting each other at C. Join DC and CE. Then CDE is the required triangle.

37. Given one angle at the base, the altitude and the perimeter of the triangle to construct the triangle. Fig. 63.

Let ∠D be the angle at the base, C the altitude and AB the perimeter.



61 62 63

Draw any line GH and take any point P in it. At P with GP make the ∠GPE = ∠D. Draw the line KL parallel to GH and at the distance C from it. Let KL cut PE at E. Take EK = EP and make KL = AB the perimeter. Join PL. Bisect PL at N; draw NF perpendicular to PL meeting KL at F. Join FP. Then PFE is the required triangle.

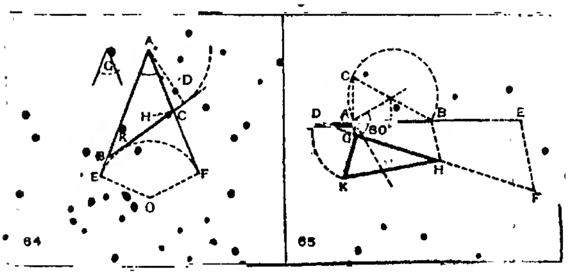
38. Given the vertical angle, altitude and half the perimeter, construct the triangle. Fig. 64.

Let ∠G be the vertical angle, AD the altitude and AF = ½ perimeter.

On AF at A make the angle FAE = ∠G. Make AE = AF = ½ perimeter. At E and F draw EO and FO perpendiculars to AE and AF respectively meeting at O. With O as centre and OE or OF as radius draw the arc EKF and with A as centre and AD as radius draw the arc DH. Draw a common tangent BKCD touching the two arcs EKF and HD at K and D. Let the tangent BKCD cut AE and AF at B and C. Then ABC is the required triangle.

39. The perimeter of a triangle is 3¼ inches, the vertical angle is 60° and one of the sides is half the base. Construct the triangle. Fig. 65.

Take any straight line AB, and draw a segment of a circle on it which will contain 60°. With A as centre and half of AB as radius intersect the arc of the segment in C. Join AC and CB. Then ACB is a triangle which has 60° vertical angle



and AC one side equal to half the base. Produce AB both ways. On the produced portions take AD = AC and BE = BC Then DE is equal to the perimeter of the triangle ACB.

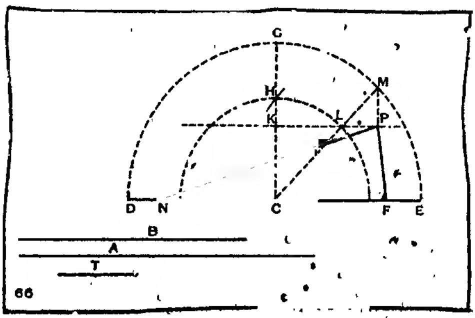
Draw DF equal to $3\frac{1}{4}$ inches at any angle with DE. Join EF and draw BH and AG through the points B and A parallel to EF. With G and H as centres and with GD and FH respectively as radii draw arcs cutting each other in K. Join KG and KH. Then KGH is the required triangle.

40. Construct a triangle having given the base, altitude and the sum of the two sides. Fig. 36.

Let B be the given base. A the sum of the two sides and T the given altitude.

Draw FN = B and bisect FN in C and produce it both ways to D and E making CD = CE = $\frac{A}{2}$ = half of the sum of the two sides. Draw CG perpendicular to FN. With F as centre and CE as radius describe an arc intersecting CG in H. Make CK = T the given altitude; through K draw KLP parallel to FN. With C as centre and CE and CH as radii draw circles. Let the smaller circle cut the line KLP in L. Join CL and produce it to meet the outer circle in M. From M draw MP parallel to CG meeting KLP in P. Join PN and PF then PNF is the required triangle.

N. B. The point P is on an ellipse of major axis DE and minor axis CH. (see chap. XII, Fig. 201).



41. To construct a triangle having given the base =1", perimeter=$2\frac{1}{2}$" and the area=0·42 Sqr.in. AB=base, EF=perimeter−base=$1\frac{1}{2}$". Fig. 67.

From the area we are to find the altitude of the triangle. Draw AB=1" and BC perpendicular to AB and equal to 0·42" Then the rectangle contained by AB and BC has the area= ·42 sqr. in. If BC is produced to D making CD=BC then BD is the altitude of the triangle on base AB with an area of ·42 sqr. inches. The problem is, now, resolved to given base, altitude and the perimeter (base+sum of two sides) of a triangle to draw the triangle which is done by prob. 40.

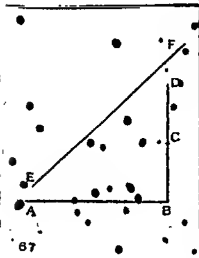
42. On a given straight line to draw a square. Fig. 68.

Let AB be the given straight line. At A on AB erect a perpendicular AC equal to AB. With B and C as centres and with radius AB draw arcs intersecting at D. Join DC and DB. Then ABDC is the square.

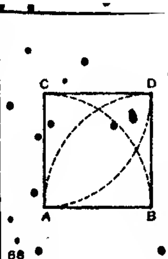
43. To construct a rectangle of given sides. Fig 69.

Let AC be a long side of the rectangle and B the length of a short side of it. At A on AC erect a perpendicular equal to B. With D and C as centres and with AC and B respectively as radii draw arcs intersecting at E. Join DE and EC. Then ACED is the required rectangle.

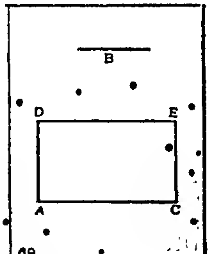
44. To construct a rectangle, the diagonal and one side being given. Fig. 70.



67



68



69

Let AB be the diagonal and C one side of the rectangle. Describe a semicircle on AB. With A as centre and radius equal to C draw an arc intersecting the semicircle in D. Join AD, DB. From A draw AE parallel to DB and from B draw BE parallel to AD meeting AE in E. Then ADBE is the required rectangle.

45. To construct a rhombus with sides equal to the given line A and an angle equal to the given angle B. Fig. 71.

Draw a straight line DE equal to A. At D on DE make the angle EDG equal to B. Make DG equal to A. With G and E as centres and with A as radius draw arcs intersecting in F. Join GF and FE. Then DEFG is the required rhombus.

46. To construct a rhomboid (parallelogram), the diagonal and the two sides being given. Fig. 72.

Let AB be the diagonal and C and D the two sides. With radius C and centres A and B describe two arcs. With radius D and from the same centres intersect the arcs in E and F. Join AE, EB, FB and FA. Then AEBF is the required rhomboid.

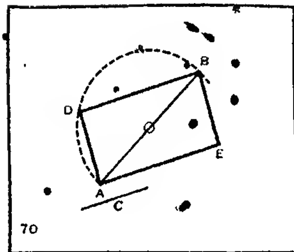
47. On a given line to construct a rectilineal figure similar to a given rectilineal figure Fig. 73.

Let ABCDE be the given rectilineal figure and FG the given line. Join AD and BD dividing ABCDE into 3 triangles. On FG make the triangle FHG similar to ABD and on the line HG make the triangle HKG similar to the triangle BCD and on the line FH make the triangle HMF similar to triangle ADE (Prop. 23, Chap V). Then the figure FGKHM is similar to the figure ABCDE and drawn on the line FG.

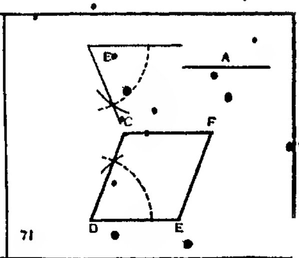
48. To construct a triangle having given A, B and C the lengths of the three medians. Fig. 74.

Let A, B and C be the three medians that is lines drawn from the vertices of a triangle to the middle points of the opposite sides.

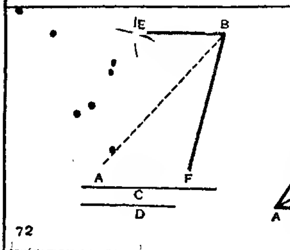
Draw D E equal to A and produce it to F such that EF $=$ EG $= \frac{A}{3}$. With F and G as centres and $\frac{2}{3}$ C and $\frac{2}{3}$ B as radii respectively describe two arcs intersecting at H. Join HE and produce it to K making EK $=$ EH. Join DH and DK. Then DHK is the triangle required.



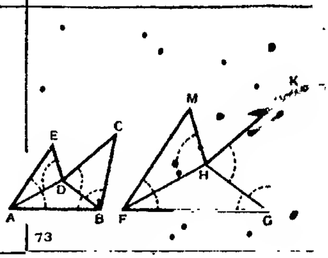
70



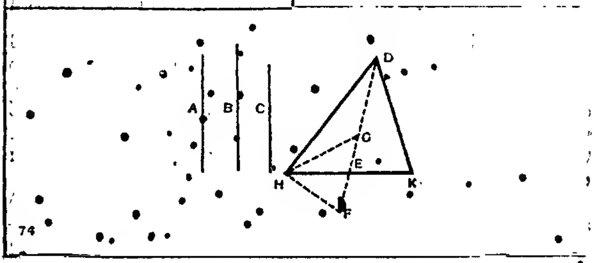
71



72



73



74

CHAPTER VI.

POLYGONS.

49. To inscribe any regular polygon in a given circle say a heptagon. Fig. 75.

Let ABC be the given circle. Draw any diameter of the circle as AB. Divide the diameter AB into the same number of equal parts as the inscribed figure has sides (in this case 7). With A and B as centres and AB as radius draw arcs intersecting at D. Join D with the 2nd point from one end and produce it to meet the circle at C. Then the chord AC is a side of the inscribed figure. Set it off seven times round the circumference and join the points.

Note—This method is very near approximation. The greatest care must be exercised in dividing the line and in drawing the line from D exactly through the point 2.

50. To inscribe any regular polygon in a given circle 2nd method (a pentagon for example). Fig. 76.

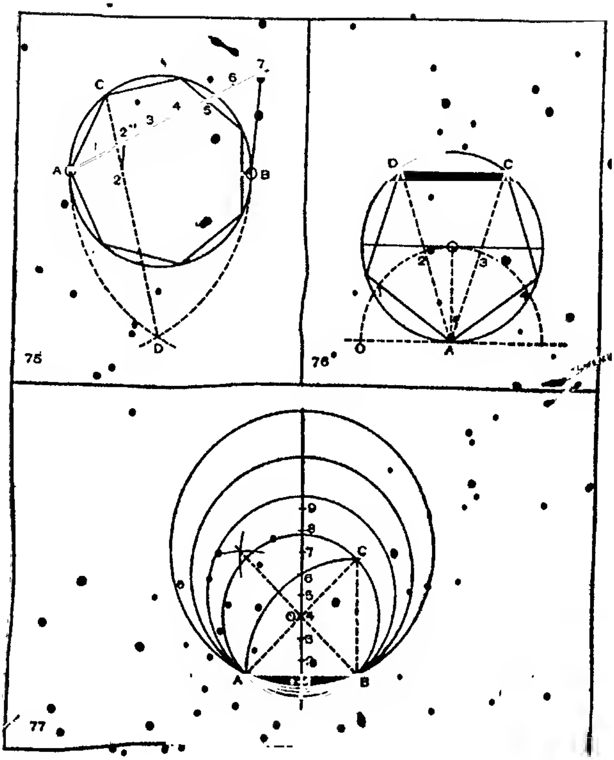
Let ACD be the given circle and draw a radius BA in it. At A draw a tangent to the circle and with A as centre and AB as radius draw a semicircle OB5. Divide the semicircle into as many equal parts as the inscribed figure has sides, in this case five. From the point A draw lines through each of these divisions till they meet the circumference of the circle ACD. Join these points which will give the required polygon.

Note.—Semicircle OB 5 gives an arc for trial division instead of the original circle

51. On a given line to describe a regular polygon (general rule). Fig. 77.

Let AB be the given line. Draw BC at right angles to AB and equal to it. Bisect AB at D. Draw DO perpendicular to AB. Bisect the right angle ABC by BO meeting DO at O. Produce DO upwards and downwards. Draw the quadrant A 6 C cutting DO produced at 6. Bisect O 6 at 5. Take the distance O 5 and set it off as 67, 78, 89 upwards of 6 and 43, 32, 21 downwards from O.

If a circle is described with O or 4 as centre and OB or OA as radius it will be seen that AB can be set off exactly 4 times in the circumference. If 5 be taken as centre and with 5A or 5B as radius a circle is described it will be seen that AB will lie 5 times on the circumference. Similarly if 6 is used as centre and 6B as radius a hexagon can be described of side AB, if 7 is used as centre and 7 B as radius a heptagon can be described of side AB so on.

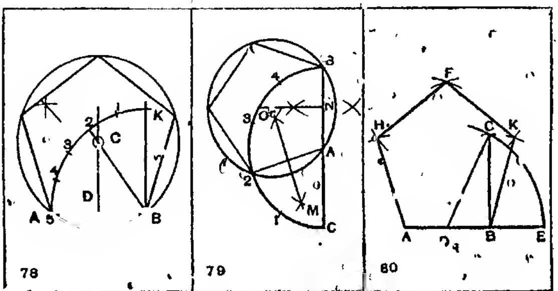

75
76
77

2nd Method. Fig. 78.—Let AB be the given line. Draw BK at right angles to AB and make it equal to AB. Draw the quadrant AK and divide it into as many equal parts as there are sides to the polygon to be described on AB (in this case five). Join B with the 2nd division from K. Bisect AB at D and draw DC perpendicular to AB meeting B2 in C. With C as centre and CB or CA as radius draw a circle. From the point A mark off round the circumference the distance AB, a side of the polygon. AB can be set off exactly five times on the circumference. Join the points and the required pentagon will be drawn.

3rd Method. Fig. 79.—Let AB be the given line. Produce BA to C making AC equal to AB. Draw a semicircle on BC the whole line. Divide the semicircle into 5 equal parts i.e., the number of sides of the polygon. Join the 2nd division from C to A as A2. Bisect AB and A2 by lines ON and OM respectively, meeting at O. With O as centre and OA or OB as radius describe a circle. Set off AB three times more on the circumference from B. It will be seen that the last point will coincide with 2 The required polygon is obtained by joining the points on the circumference.

52. On a given line to construct a regular pentagon (special method). Fig. 80.

Let AB be the given line. At B draw BC perpendicular to AB and equal to it. Bisect AB in D. With D as centre and DC as radius draw the arc CE meeting AB produced in E. With A and B as centres and AE as radius draw arcs intersecting



in F. With A, B and F as centres and radius equal to AB draw arcs intersecting at H and K. Join AH, HF, FK and KB. Then ABKFH is the required pentagon.

53 On a given line to construct a regular hexagon. Fig. 81.

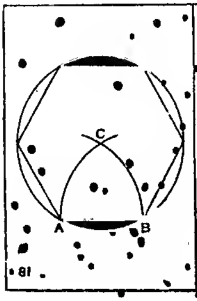
Let AB be the given line. With A and B as centres and radius AB draw arcs intersecting each other at C. With C as centre and radius AB draw a circle On the circumference of this circle set off the length of the radius all round from A; the points being joined will give the required hexagon.

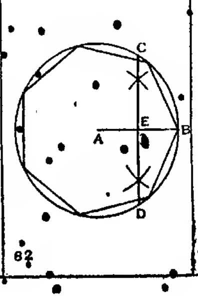
54 In a given circle to inscribe a regular heptagon (special method). Fig. 82.

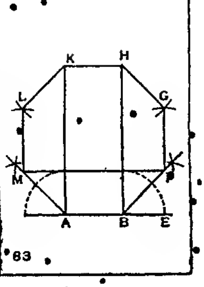
Let DBC be the given circle. Draw any radius AB in it and bisect it in E Through E draw CED perpendicular to AB meeting the circle in C and D. Then CE or ED is the length of one side of the heptagon to be inscribed in the circle. Set off this distance round the circle and join the points to get the required heptagon.

55. On a given line AB to construct a regular octagon. Fig. 83.

At A and B draw perpendiculars AK and BH Produce AB to E and bisect the angle HBE by the line BF. Make BF = AB Similarly produce BA and bisect the outer right angle at A by the line AM and make AM = AB. Join MF. Make the two perpendiculars AK and BH each equal to MF. From the points K, H, F and M as centres and with AB as radius draw arcs intersecting at L and G. Join ML, LK, KH HG, GF. The figure ABFGHKLM is the required octagon.







The same, 2nd method :—(Fig. 84).

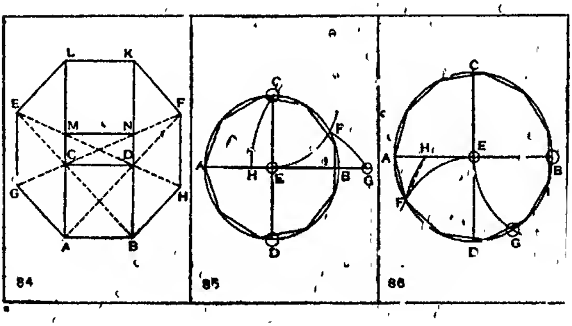
On AB draw a square ABDC. Draw the diagonals BC and AD and produce them upwards to E and F. Make CE and DF each equal to AB. Through the points A and F, B and E draw lines parallel to BE and AF respectively. Make these parallel lines equal to AB and join GE, FK and FH. Then the octagon is completed.

56. In a given circle to inscribe a regular nonagon. Fig. 85.

Let ADC be the given circle. Draw the diameters AB and CD perpendicular to each other. With C as centre and CE as radius draw the arc EF cutting the circle in F. With D as centre and DF as radius draw an arc FG cutting AB produced in G. With G as centre and GC as radius draw the arc CH cutting AB in H. Then HA is a side of the nonagon to be inscribed in the circle. Set it off nine times on the circumference and by joining the points the nonagon is obtained.

57. In a given circle to inscribe a regular undecagon. Fig. 86.

Let A3D be the given circle. Draw the two diameters AB and CD at right angles to each other and intersecting in E. With D as centre and DE as radius draw an arc EF cutting the circle in F. With B as centre and BE as radius draw an arc cutting the circle in G. With G as centre and GF as radius draw an arc FH cutting the diameter AB in H. Join the chord FH Then the chord FH is equal to one side of the undecagon.



58. In a given circle to inscribe a regular quindecagon. Fig. 87.

Let ABC be the given circle. Inscribe an equilateral triangle ABC in the circle. Then the circumference is trisected. Inscribe a regular pentagon in the circle ABC with a vertex at A as ADEFG. The pentagon divides the circumference in 5 equal parts. The arc DB = arc AB − arc AD = $(\frac{1}{3} - \frac{1}{5})$ of the circumference = $\frac{2}{15}$ circumference. Bisect the arc DB in H. Then the chord DH is a side of the quindecagon.

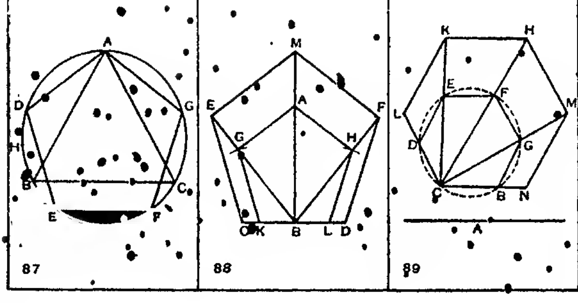
59. To construct any regular polygon the length of the diameter being given (say a pentagon.) Fig. 88.

Let AB be the given diameter. Through B draw a line at right angles to AB and make BC = BD on each side of B. On CD describe a regular pentagon CEMFD. Join BE, BM and BF. From A draw AG and AH parallel to ME and MF meeting BE and BF in G and H. Draw GK and HL parallel to EC and FD meeting CD in K and L. Then AGKLH is the required pentagon.

60. To construct any regular polygon, the length of any diagonal being given. Fig. 89.

Let A be the length of one of the longer diagonals of a regular hexagon.

On any base CB construct a regular hexagon CDEFGB. From C draw the diagonals CE, CF and CG. Produce CF the longer diagonal to H making CH equal to A. Produce the other diagonals and from H draw HK and HM parallel to FE and FG meeting the diagonals CE and CG produced in K and M. From K and M draw KL and MN parallel to ED and GB meeting CD and CB produced in L and N then CLKHMN is the requird hexagon



CHAPTER VII.

TO INSCRIBE AND CIRCUMSCRIBE RECTILINEAL FIGURES.

61. To inscribe an equilateral triangle in a given square. Fig. 90.

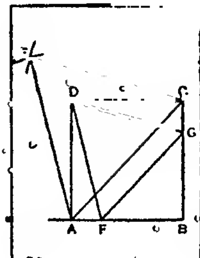
Let ABCD be the given square. Draw AC a diagonal of the square. On AC describe an equilateral triangle CEA From D a corner of the square within the triangle draw DF and DG parallel to EA and EC respectively and meeting the sides of the square in F and G. Join GF. Then DGF is the equilateral triangle.

62. To inscribe an equilateral triangle in a given pentagon. Fig. 91.

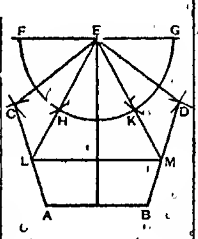
Let ABDEC be the given pentagon. Through E draw FG parallel to AB. With E as centre and with any radius draw a semicircle FHKG. From F and G as centres and with the same radius intersect the semicircle in H and K. Join EH and EK and produce them to meet the sides CA and DB in L and M. Join LM. Then ELM is the equilateral triangle

63. To describe an equilateral triangle about a given square. Fig. 92.

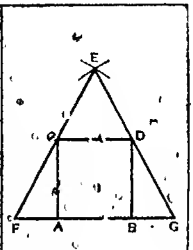
Let ABCD be the given square. On CD the top line of the square draw an equilateral triangle CED. Produce EC and ED to meet AB produced in F and G. Then EFG is the equilateral triangle.



90



91



92

64. In a given triangle to inscribe a square. Fig. 93.

Let ABC be the triangle. Draw AE perpendicular to BC the base. Draw AD at right angles to AE and equal to it. Join BD cutting AC in G. From G draw GK parallel to AE meeting BC in K and GF parallel to BC meeting AB in F. Draw FH parallel to AE. Then FGKH is the required square.

Note:—The method of construction of problems 61 and 64 is similar. It is by the locus of points of similar figures. In fig 93 the smallest square may be imagined to be in the corner B and the biggest square is on the line AE so that by joining B and D the locus of a corner of the square is found which intersecting the side AC gives the position of one corner of the square to be inscribed in the triangle.

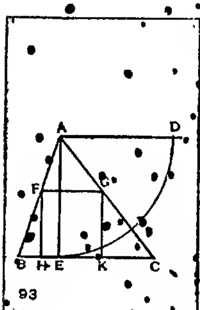
65 In a given triangle to inscribe a rectangle having one of its sides equal to a given line. Fig. 94.

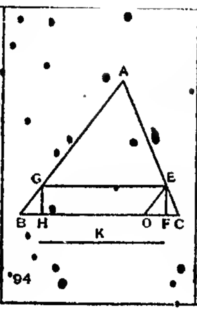
Let ABC be the given triangle and K the given line.

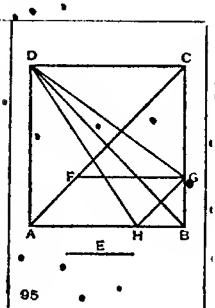
From B along the base BC of the triangle ABC measure BD equal to K. Draw DE parallel to BA meeting AC in E. From E draw EG parallel to BC. From G and E draw GH and EF perpendicular to BC. Then GEFH is the required rectangle.

66. To inscribe an isosceles triangle in a given square having a base equal to a given line Fig. 95.

Let ABCD be the given square and E the given base. Draw AC the diagonal of the square and on AC set off AF equal to E. Draw FG parallel to AB and GH parallel to AC. Join DH and DG. Then DHG is the required isosceles triangle.







67. To inscribe a square in a given quadrilateral figure which has its adjacent pairs of sides equal. Fig. 96.

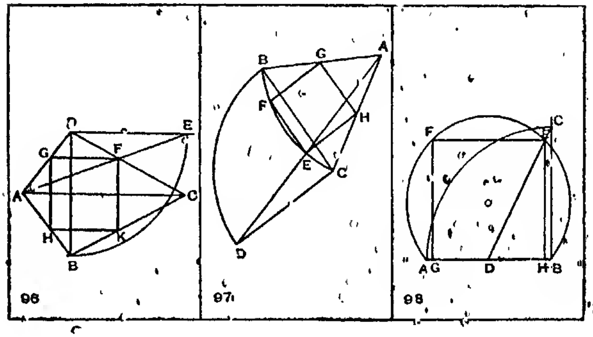
Let ABCD be the given quadrilateral. Draw the two diagonals AC and BD. From the extremity D of the diagonal BD draw DE at right angles to DB and equal to it. Join AE cutting DC in F. Draw FG parallel to AC meeting AD in G. From the points F and G draw FK and GH parallel to DB meeting the sides in K and H. Join HK. Then FGHK is the required figure. The construction is similar to that of problem 62.

68. To inscribe a square in a sector. Fig. 97.

Let ABC be the sector. Join BC the chord. Draw CD at right angles to CB and equal to it. Join DA cutting the arc BC in E. Draw EF parallel to BC meeting the arc in F. Draw FG and EH at right angles to FE from the points F and E meeting the sides in G and H. Join GH. Then EFGH is the required square.

69. To inscribe a square in a segment. Fig. 98.

Let AFEB be a segment of a circle. Draw BC at right angles to AB and equal to it. Bisect AB at D. Join CD cutting the arc in E. Draw EF parallel to AB meeting the arc in F. From E and F draw EH and FG parallel to BC meeting AB in H and G. Then EFGH is the square.



70. To inscribe a square within another square having a side equal to a given length. Fig. 99.

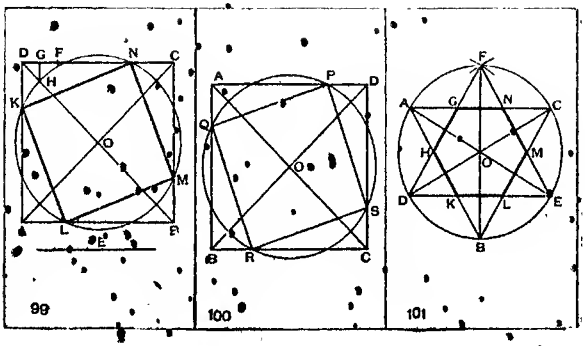
Let ABCD be the given square and E the given length. Make CF equal to E. Bisect DF in G. Draw GH parallel to DA meeting the diagonal DB in H. With O as centre and OH as radius draw a circle cutting the sides of the square in K, L, M and N. Join KLMN, which is the required square.

71. To inscribe a square in another square one corner of the inscribed figure to be in a given point. Fig 100.

Let ABCD be the given square and P a given point in AD. Draw AC, BD the diagonals intersecting in O. With O as centre and radius OP describe a circle cutting the sides of the square in 8 points. Join every third point from P and the square PQRS is obtained.

72. To inscribe a regular hexagon in an equilateral triangle. Fig. 101.

Let FDE be the equilateral triangle. Bisect each angle of the triangle by the lines FB, DC and EA intersecting in O. With O as centre and OF as radius describe a circle cutting the bisectors in B, C and A. Join AB, BC, CA cutting the sides of FDE in H, K L, M, N, and G Join HK LM and NG. The figure GNMLKH is the hexagon.



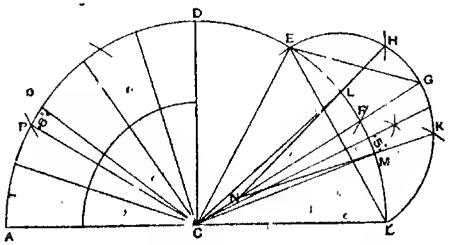
CHAPTER VIII.

CONSTRUCTION OF ANGLES AND PROTRACTORS.

Plane angles are obtained by the revolution of a straight line from a fixed straight line about a point where the two lines meet. It is measured first by dividing a complete revolution or the circle described by the revolving line into 4 equal parts by drawing two diameters in the circle at right angles to each other. Each portion is called a right angle which is a quadrant of the circle. The unit of angular measurement is obtained by dividing a right angle into 90 equal parts. Each part is called a degree. A degree is subdivided into 60 equal parts called minutes and a minute is subdivided into 60 equal parts called a second.

Instead of dividing a right angle into 90 equal parts which in circles up to 4 inches radius is almost impracticable on paper a degree can be obtained in the following way.

Take a straight line AB say 4 inches long. Draw a semicircle on AB and let C be the centre of the circle. Draw CD perpendicular to AB. Trisect the angle DCB by CE and CF. Then each of the small angles is 30°. Trisect the angle ECB or 60° by the method given below applicable to all acute angles. (Fig 102)



102

Trisecting an acute angle ECB :—Join EB the chord and draw EGB a semicircle on EB. Produce CF, the bisector of the angle, to bisect the semicircle in G. Join EG, take FN in FC equal to EG. Trisect the semicircle EGB in H and K. Join NH and NK intersecting the arc EB in L and M respectively. Join CL and CM which trisect the angle ECB. Therefore

$\angle$BCM is 20° But $\angle$BCF $=30^{\circ}$. Therefore $\angle$FCM $=\angle$BCF $-\angle$BCM $=30^{\circ}-20^{\circ}=10^{\circ}$. If $\angle$FCM is bisected an angle of 5° will be obtained. An angle of 6° can be obtained thus: Divide the right angle ACD on the left into 5 equal parts by Prob. 19. Fig 45.

Then each part is 18°. Divide the quadrant into 3 equal parts then each part is 30° as $\angle$ACP. If this angle be subtracted from the twice 18° divisions as $\angle$ACb the difference $\angle$pcb is $36^{\circ}-30^{\circ}=6^{\circ}$

If half of the $\angle$FCM be subtracted from $\angle$pcb 1° is obtained. Fig. 102.

The interior angles and sides of regular polygons :—The interior angle of a regular polygon is found by dividing 360° by the number of sides of the polygon which gives the angle at the centre subtended by a side of the polygon and subtracting this angle at the centre from 180°.

When the interior angle of a regular polygon is given the number of sides of the polygon is found by dividing 360° by the difference of the interior angle from 180°. This difference is the angle at the centre.

Names of polygons		Angles subtended at the centre.	Interior angles.
Triangle	...	$\frac{360}{3}=120^{\circ}$;	$180^{\circ}\text{-}120^{\circ}=60^{\circ}$
Square	...	$\frac{360}{4}=90^{\circ}$;	$180^{\circ}\text{-}90^{\circ}=90^{\circ}$
Pentagon	...	$\frac{360}{5}=72^{\circ}$;	$180^{\circ}\text{-}72^{\circ}=108^{\circ}$
Hexagon	...	$\frac{360}{6}=60^{\circ}$;	$180\text{-}60^{\circ}=120^{\circ}$
Heptagon	...	$\frac{360}{7}=51^{\circ}\frac{3}{7}$;	$180\text{-}51\frac{3}{7}=128\frac{4}{7}$
Octagon	...	$\frac{360}{8}=45^{\circ}$;	$180^{\circ}\text{-}45^{\circ}=135^{\circ}$
Nonagon	...	$\frac{360}{9}=40^{\circ}$;	$180^{\circ}\text{-}40^{\circ}=140^{\circ}$
Decagon	...	$\frac{360}{10}=36^{\circ}$;	$180^{\circ}\text{-}36^{\circ}=144^{\circ}$
Undecagon	...	$\frac{360}{11}=32\frac{8}{11}$;	$180^{\circ}\text{-}32\frac{8}{11}=147\frac{3}{11}$
Duodecagon	...	$\frac{360}{12}=30^{\circ}$;	$180^{\circ}\text{-}30^{\circ}=150^{\circ}$.

A protractor is an instrument used for measuring or setting off angles. It is either circular, semicircular or more commonly rectangular in shape. A variety of scales are drawn on both sides of the rectangular protractor which are very convenient. Circular protractors are usually made of card boards about 1 foot to 15 inches in diameter and the circumference is divided into degrees, half degrees and quarter degrees i. e., up to 15 minutes. This protractor is very convenient for plotting the bearings of a survey.

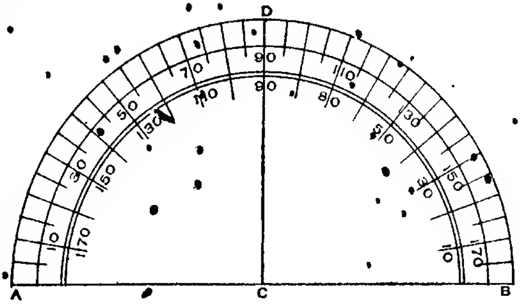
Semicircular protractors are usually made of brass and the arc is divided into degrees only. It is found in cheap drawing boxes and figure 103 is a sketch of it. Point C is the centre of the circle from which the radiating lines are drawn and AB is the diameter. Fig. 103.

The most common protractor is of rectangular form 6″ long and $1\frac{3}{4}$″ wide and is shown in fig. 104. The degrees are numbered in the primary divisions equal to 10° each in the middle place from left to right and marked by lines drawn through the 3 spaces. Each of these primary divisions is again subdivided into 10 secondary divisions of a degree each and marked on the outer space by radial lines. The primary division of 10 degrees are again written from right to left on the 3rd space so that the protractor may be used from either end. The protractor is used by placing the edge AB to coincide with the line on which the angle is to be drawn and the middle point C against the point in the line from which the angle is to start. If the angle is less than 180° the protractor is placed either above or on the right of the line as the line is situated with reference to the paper and the required angle marked, and if the angle is over 180° it is placed below or on the left of the line and the difference of 180° from the angle is laid out. In marking the divisions on the paper care should be taken to hold the pencil or needle point quite close to the edge of the protractor.

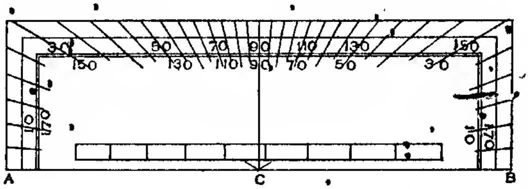
Scale of chords :—

The most convenient way to take off angles is by the protractor but there is another way of measuring angles more accurately, known as the scale of chords (fig. 105). It is found on one or both faces of rectangular protractors marked as CHO.

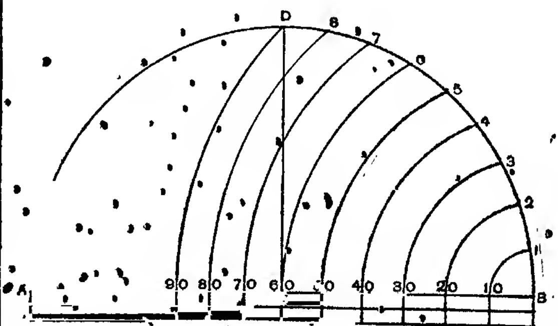
Construction of scale of chords :—Draw a line AB and bisect it at C. Draw CD perpendicular to AB. With C as centre and any assumed radius as CB draw a semicircle ADB. Divide the quadrant BCD into 9 equal parts *i. e*, each part is equal to 10° = a primary division on the protractor. Number these divisions from B to D as 1, 2, 3, 4, 5, 6, 7 and 8. With B as centre and B1, B2, B3, B4 B5 &c to BD as radii describe arcs to intersect AB in nine points. It will be seen that the arc with B6 as radius will pass through C the centre. Draw another line below AB and parallel to it and from the points of division on the line AB draw lines perpendicular to it and complete the scale. The divisions are marked as 10°, 20°, to 90° from B towards A. The spaces are all unequal and they gradually decrease in length from the first to the last. Divide each of these 10° divisions



103



104



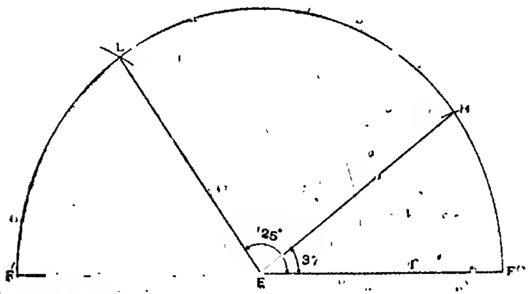
105

into 10 equal parts for the approximation of chords for intermediate angles. The radii used in dividing AB are the chords of the different arcs of 10°, 20°, &c. consequently the scale thus obtained is called the scale of chords. Fig. 105. The scale of chords may similarly be constructed for laying out angles expressed in grades by dividing the right angle or the quadrant into 10 equal parts instead of 9.

The use of scale of chords in setting off angles in degrees:—
Example:—To draw an angle of 37° and an angle of 125°. Fig. 106. Draw a straight line as EF. With E as centre and the length from 0 to 60° on the scale of chords as radius draw the arc FH. Then take the distance 0° to 37° from the scale and set it off on the arc FH from F to H. Join EH. Then ∠FEH is 37°.

The scale of chords show only chords up to 90°. To lay off 125° subtract it from 180° and the remainder 55° is to be set off from the left in the same way as 37° is laid on the right. Then the suppliment FEL is 125°.

Usually there are two scale of chords on a 6" ivory or wooden rectangular protractor one of 3" radius is put below the protractor side and the other of 2" radius is placed on the right hand top of the scale side.



106

CHAPTER IX.

CIRCLES AND CIRCLES TOUCHING RIGHT LINES AND CIRCLES.

73. Find the centre of a given circle. (Fig. 107)

Construction. (1) Draw a chord in the circle as AB. Bisect it by a line CD drawn at right angles to the chord and terminated by the circumference. It is a diameter of the circle, the point which bisects this line is the centre.

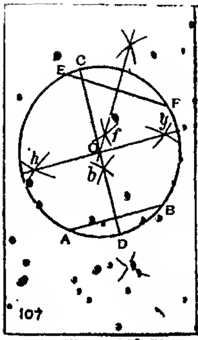
Construction. (2) Draw any two chords of the circle AB and EF. Bisect the chords and draw lines at right angles. The point where these lines intersect is the centre of the circle. (Fig. 107).

74. To draw radial lines from points on a given arc, the centre of the circle being inaccessible. (Fig. 108)

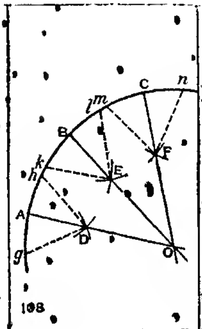
Let A B C be the three points on a given arc whose centre is not known. With centres A, B and C and with any convenient radius draw arcs cutting the given arc on each side of the given points as g, h on two sides of A, k, l on two sides of B etc. With g, h as centres and with any length as radius draw arcs intersecting at D. Join AD and produce, it will pass through the centre of the given circle. Similarly with k and l as centres draw arcs intersecting in E. Join BE which produced will cut the line AD produced at the centre of the circle.

75. To inscribe a circle in a given triangle. (Fig. 109)

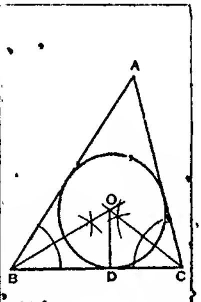
Bisect any two angles of the triangle as ABC and ACB by BO and CO which meet at O. Then O is the centre of the



107



108



109

circle. From O draw OD perpendicular to BC. With O as centre and radius OD if a circle is drawn it will touch the three sides of the triangle.

76. To draw an escribed circle tangential to one side of the triangle and the other two sides produced. (Fig. 110).

Let ABC be the triangle. Produce the two sides AB and AC to D and E. Bisect the angles DBC and BCE by BO and CO meeting in O. Draw OF perpendicular to BC. With O as centre and OF as radius draw a circle which will touch BC, BD and CE.

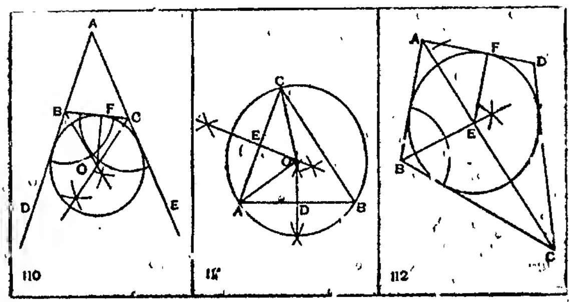
77. To describe a circle about a given triangle. (Fig 111.)

Let ABC be the triangle. Bisect any two sides as AB and AC at D and E. Draw DO and EO perpendiculars to AB and AC meeting at O. Then O is the centre of the circle. Join OA, OB, and OC which are all equal and which are the radii of the circle. With O as centre and OA or OB or OC as radius draw a circle which will pass through the three corners of the triangle.

78. To inscribe a circle in a given quadrilateral which has its adjacent pairs of sides equal. (Fig. 112.)

Let ABCD be the given quadrilateral with the adjacent pairs of sides equal.

Draw one diagonal AC. Bisect one of the opposite angles as ABC by BE meeting AC in E. From E draw a perpendicular to any one of the sides as EF on AD. With E as centre and EF as radius draw a circle which will touch the four sides of the quadrilateral.



79. To draw a tangent to a given circle. (a) from a point in the circumference, (b) from a given external point. Fig. 113.

(*a*) Let AFE be a circle and E a point in the circumference. Mark C the centre of the circle. Join CE. At E on CE erect a perpendicular as EG. Then EG touches the circle at E.

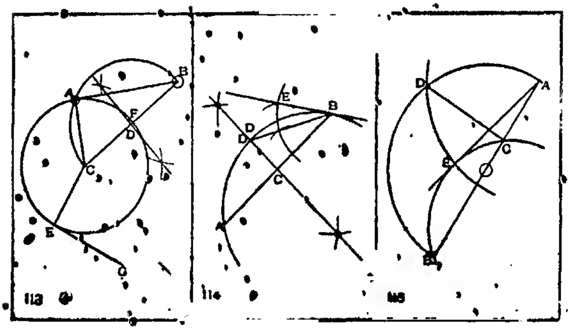
(*b*) Let B be an external point. Join BC. On BC draw a semi-circle CAB cutting the circumference in A. Join AB. Then BA is a tangent to the circle.

80 To draw a tangent to the arc of a circle at a given point in the circumference when the centre is inaccessible. Fig. 114.

Let ADB be an arc of a circle. B a point in it. Draw BA any chord of the arc. Bisect it at right angles by CD meeting the arc in D. Join BD. At B make the angle DBE equal to DBC. Then BE is a tangent to the circle.

81. To draw a tangent to an arc from a given external point when the centre is inaccessible. Fig. 115.

Let A be the given external point and BEC the arc. Draw any line ACB through A cutting the arc in C and B. On AB draw a semi-circle ADB and from C draw CD perpendicular to AB meeting the semi-circle in D. With A as centre and AD as radius draw an arc cutting the given arc BEC in E. Join AE. Then AE is a tangent to the given arc from the given point A.



82. To draw a common tangent to two unequal circles in the same direction. (or an external tangent). Fig. 116.

Let A and B be the centres of the two circles. Join A and B. From A the centre of the greater circle and with a radius equal to the difference of the radii of the two circles (AD = AC — DC but DC = BE) draw a circle DPR. Bisect AB in F and on AB draw a semi-circle meeting DPR in D. Join AD and produce it to meet the outer circle in C. Draw BE parallel to AC meeting the smaller circle in E. Join CE which will be a tangent to the two unequal circles.

83. To draw a common tangent to two unequal circles in the opposite directions (or an interior tangent). Fig. 117.

Let A and B be the centres of the two unequal circles. Join AB. From B the centre of the larger circle measure BH equal to the sum of the radii of the two circles. With B as centre and BH as radius draw a circle HC; on AB describe a semi-circle cutting the circle HC in C. Join BC cutting the circumference of the larger circle in D. Draw AE parallel to BC meeting the circumference in E. Join ED which is the interior tangent.

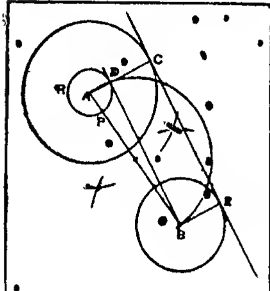
84. To draw a common tangent to two equal circles in the opposite directions. Fig. 118.

Let A and B be the centres of the two equal circles; join AB and bisect it in C. On AC draw a semi-circle cutting the circle A in D. Join AD. Draw BE parallel to AD and in the opposite direction of AD. Join DE which will pass through the bisection point C and will be an interior tangent.

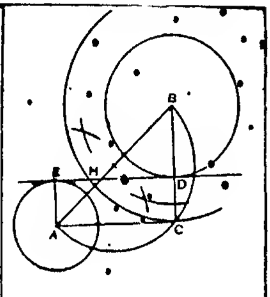
85. To describe a circle of a given radius to touch two converging straight lines Figs. 119. and 120.

(*a*) Let AB, BC be the two converging lines. If the two lines meet at B then bisect the angle ABC by BE; draw FH parallel to AB and at a distance equal to the given radius R. FH intersects BE in E which is the centre of the circle. From E draw EK perpendicular to AB Whith E as centre and EK as radius draw a circle which will touch AB and BC. Fig. 129.

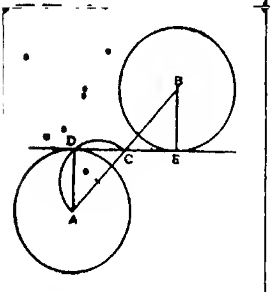
(*b*) If AB and bC do not meet, draw FH and DG parallel to AB and bC respectively at the same distance R the given radius. The two lines intersect at E which is the centre of the circle. This construction is applicable to case (*a*) also. Fig 120.



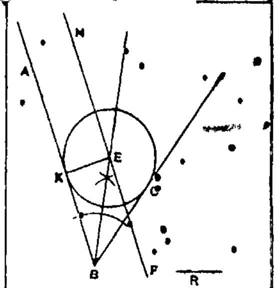
116



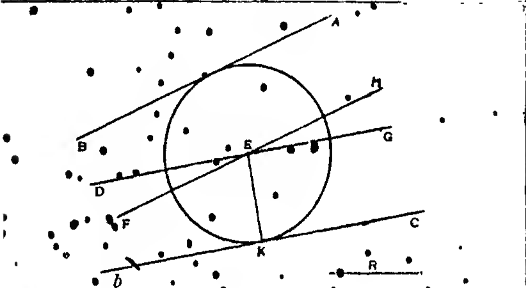
117



118



119



120

86. To describe a circle to touch two converging lines and passing through a given point P within the angle. Fig. 121.

Let BAC be the given angle and P a point within it. Bisect the angle BAC by AE. Centres of all circles within the angle and touching the two lines lie on AE. Take a point D in AE and draw DF perpendicular to CA. With D as centre and DF as radius draw a circle which will touch AB and AC. Join AP cutting the arc of the circle centre D at G. Join DG. Draw PH parallel to DG meeting AE in H. From H draw HB perpendicular to BA. Then H is the centre and HB is the radius of the circle which will touch the two lines and pass through P.

87. To describe a circle to touch two converging lines and touching one in a given point P. Fig. 122.

Let AB and CD be the two converging lines. Find EF the bisector of the angle made by AB and CD by fig. 41, ch. iv. P is a point in DC. From P draw PF perpendicular to DC meeting the bisecting line EF in F. Then F is the centre of the circle and FP is the radius.

88. To draw a circle to pass through two given points P and Q and to touch a given straight line AB. Figs. 123 and 124.

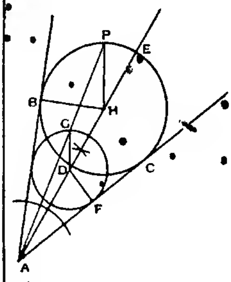
(*a*) Let P and Q be the two given points and AB a given line not parallel to the line PQ. Join PQ and produce it to meet AB produced in D. Bisect PQ in C.

With C as centre and CD as radius draw an arc of a circle. From P draw PF perpendicular to PD to meet the arc in F. Take DE = PF. From C and E draw perpendiculars to QP and AB to meet at O. Then with O as centre and OE as radius draw a circle QPE which will pass through the two points Q and P and touch AB in E. Fig. 123.

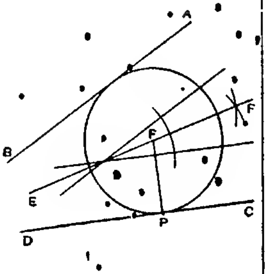
(*b*) If AB be parallel to the line PQ then join PQ and bisect it at D. From D draw DC perpendicular to QP meeting AB in C. Join QC Bisect QC at right angles by EO meeting DC in O. Then O is the centre and OC or OQ is the radius of of the circle. Fig. 124.

89. To describe a circle of any given radius to touch a line and pass through a given point P. Fig. 125.

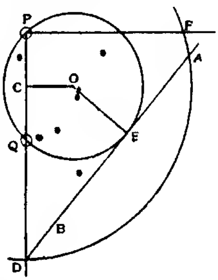
Let AB be the given line. R the given radius and P the given point. Draw CD parallel to AB at a distance R. With P as centre and radius R draw an arc cutting CD in O. Then O is the centre of the circle and R the radius.



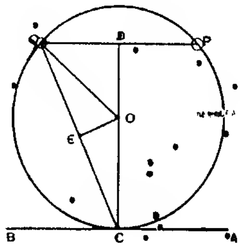
121



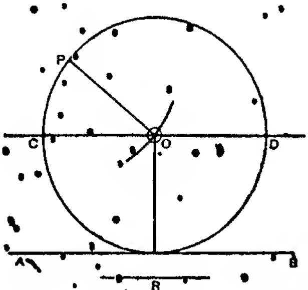
122



123



124



125

90. To describe a succession of three circles touching two converging lines and each other. Fig. 126.

Let BAC be two converging lines. Bisect the angle BAC by AD. Take any point E in AD and draw EF perpendicular to AC. With E as centre and EF as radius draw a circle cutting bisecting line AD in G and H. Through G and H draw lines perpendicular to AD meeting AB and AC in L N and K M. Bisect the angle AKL by KQ meeting AD in Q. From Q draw QR perpendicular to AB, Then Q is the centre and QR is the radius of a circle which will touch the two lines and the circle. Similarly bisect the angle NMC by MP and draw PS perpendicular to AB. Then P is the centre and PS is the radius of the 3rd circle.

2nd method.—

With L as centre and LG as radius draw an arc cutting AB in R. From R draw RQ perpendicular to AB meeting AD in Q. Then Q is the centre of one circle.

Similarly with N as centre and NH as radius draw an arc cutting AB in S. From S draw SP perpendicular to AB. Then P is the 3rd centre.

91. To describe a circle of a given radius to touch a given straight line and a given circle. Fig. 127.

Let R be the given radius, AB the given straight line and FG the given circle. This circle can not be at a distance of more than 2R from AB.

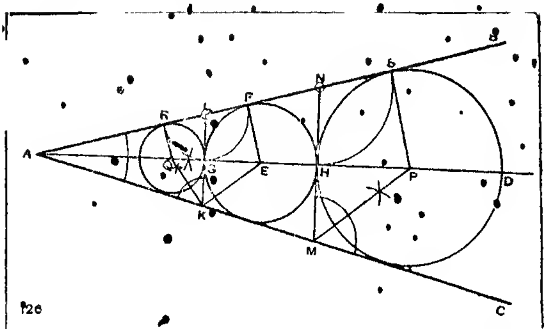
Draw CD parallel to AB at distance R the given radius. Take O as the centre of the given circle. With O as centre and radius equal to the sum of the radii of the two circles (*i. e.* R and OF, the radius of the given circle) draw an arc intersecting CD in E. Then E is the centre and R the radius of the required circle.

92. To describe a circle to touch a given line AB and a given circle centre C in a point P. Fig. 128.

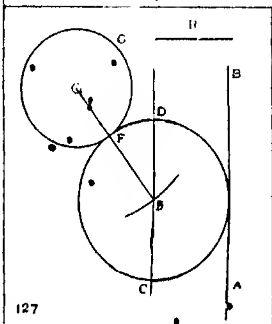
Join CP and produce it. Through P draw a tangent to the circle meeting AB in D. Bisect the angle PDA by DE meeting CP produced in E. Then E is the centre of the circle and EP is the radius of the circle to be described.

93. To describe a circle to touch a given circle O and a given line AB in a point P. Fig. 129.

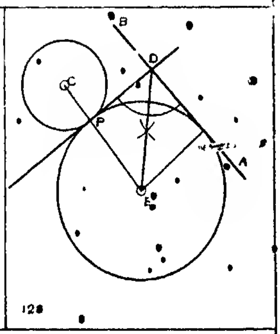
Through C centre of circle C and P draw lines CD and PG perpendiculars to AB. Produce DC to meet the circum-



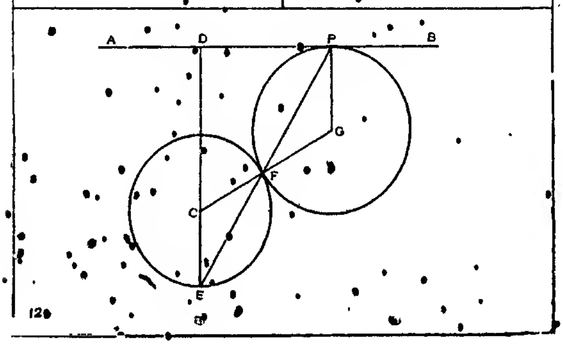
126



127



128



129

ference of the circle in E. Join PE cutting the circle C in F. Join CF and produce it to meet PG in G. Then G is the centre and GP is the radius of the circle required.

94. To describe a circle of a given radius tangential to two given unequal circles (a) internally (b) externally Figs. 130 and 131.

(*a*) Let A and B be two unequal circles whose centres are A and B. Let R be a given radius. Join AB cutting the circles A and B in C and D respectively. Produce AB both ways and take CF and DE each equal to R on the outside of C and D. With A as centre and AF as radius draw an arc and with B as centre and BE as radius draw another arc intersecting he first in O. Then O is the centre and R is the radius and a circle is drawn it will touch A and B and include them. Fig. 130.

(*b*) Take CF and DE each equal to R on the inside of C and D and follow the direction as given in (*a*). In this case the circle drawn from O with radius R will touch the two given circles externally. Fig. 131.

95. To describe a circle tangential to and including two given unequal circles and touching one of them in a given point C. Fig. 132.

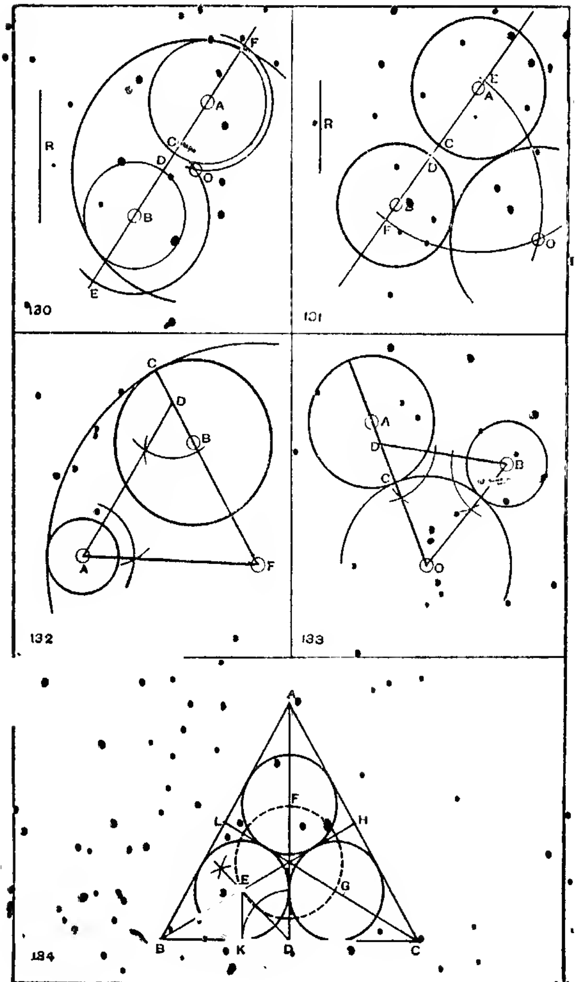
Let A and B be the given circles and C a point in B, the larger circle. Let A and B be the centres of the two circles. Join CB. From C measure CD in CB equal to the radius of the smaller circle. Join AD. At A in AD make the angle DAF equal to the angle ADB meeting CB produced in F. Then with F as centre and FC as radius draw a circle which will touch A and touch B in C and include them.

96. To draw a circle tangential to two given unequal circles externally and touching one of them in a point C Fig. 133.

Find A and B the centres of the circles. Let C be a point in the circumference of the larger circle. Join CA. Cut off CD from CA equal to the radius of the smaller circle. Join DB. At B in BD make an angle DBO equal to BDC. Produce DC to meet BO in O. With O as centre and radius OC draw a circle which will touch the other circles.

97. To inscribe three equal circles in a given equilateral triangle, each circle to touch the other two as well as two sides of the given triangle. Fig. 134.

Let ABC be the equilateral triangle. Bisect each of the


130
131
132
133
134

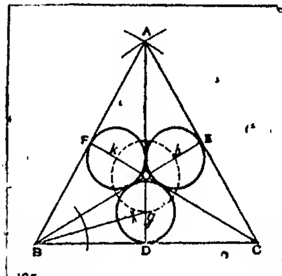
angles A, B and C by AD, BH and CL respectively. Bisect the angle ADB by DE meeting BH in E. Then E is the centre of one circle. Take CG and AF each equal to BE. Then G and F are the remaining two centres. From E draw EK perpendicular to BC. Then EK is the radius of each circle.

98. To inscribe three equal circles in an equilateral triangle, each touching one side and two circles. Fig. 135.

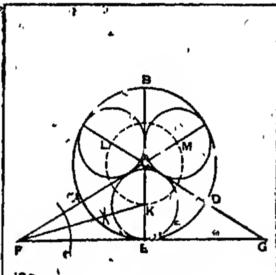
Bisect the three angles A, B and C of the triangle ABC by AD, BE and CF meeting one another in O. Bisect the angle OBC by the line BG meeting OD in g. With g as centre and gD as radius draw a circle. With O as centre and Og as radius draw a circle intersecting OF and OE in k and h respectively. Then h and k are centres of the other two circles.

99. In a given circle to draw three or more equal circles touching each other. Fig. 136.

Find the centre C of the circle. Divide the circumference into 3 equal parts or as many equal parts as the number of circles to be inscribed. Draw the three radii CA, CD and CB dividing the circle into 3 equal parts. Bisect the angle ACD by CE meeting the circle in E. Through E draw a tangent till it meets the line CA and CD produced in F and G. Bisect the angle CFE by FK to meet CE in K. With C as centre and CK as radius draw a circle cutting the bisectors of the angles ACB and BCD in L and M respectively. With K, L, and M as centres and radius equal to KE draw three circles which will touch each other and the outer circle.



135



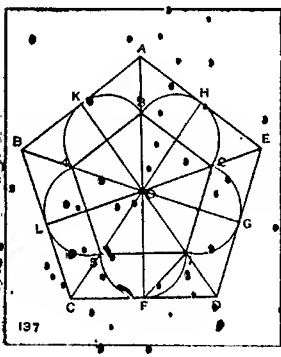
136

100. To inscribe within any regular polygon as many semicircles as the figure has sides, each touching one side and having their diameters adjacent. Fig. 137.

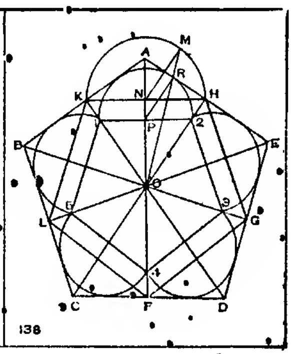
Let ABCDE be the given regular pentagon. Bisect the angles A, B, C, D, E by AF, BG, GH, DK and EL cutting one another at O. Bisect the angle AFD by F1 meeting OD in 1. This point 1 gives us a point of the inner pentagon on which the semicircles are to be described. From point 1 draw 12 parallel to DE, meeting OE in 2. Complete the pentagon 12345, describe semi-circles on 12, 23, 34, 45 and 51. They will touch the sides DE, EA, AB, BC and CD.

101. To inscribe within any regular polygon as many semicircles as the figure has sides, each touching two sides and having their diameters adjacent. Fig. 138

Let ABCDE be a regular polygon (in this case a pentagon.) Bisect each of the angles A, B, C, D, E of the polygon by lines AF, BG, CH, DK and EL intersecting one another in O, the centre of the pentagon and meeting the opposite sides in F, G, H, K and L Join FG, GH, HK, KL and LF. On one of these lines say KH draw a semi-circle KMH. Let N be the middle point of KH the base of the semi-circle. From N draw NM perpendicular to AE meeting the semi-circle in M. Join MO cutting the side AE in R. From R draw RP parallel to MN meeting AO in P. Through P draw a straight line 1P2



137



138

parallel to KH meeting OK and OH in 1 and 2. similarly from 2 draw 23 parallel to HG, from 3 draw 34 parallel to GF &c. On 12, 23, 34 &c. draw semi-circles which will touch the two sides of the polygon and have adjacent diameters.

The inscribed figure is a foiled figure required in tracery works. These foiled figures have adjacent diameters. There are foiled figures with tangential arcs, a case of which is shown in the next problem.

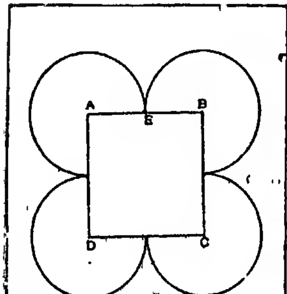
102. To construct a foiled figure about any regular polygon, having tangential arcs say a square, Fig. 139

Let ABCD be a square. Bisect one side AB in E. With A, B, C, and D as centres and radius AE draw the 4 arcs up to the sides of the square.

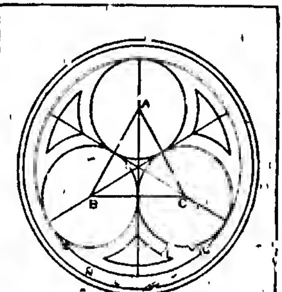
It will be seen that a side of the polygon is equal to double the radius of one foil.

103. To draw a Gothic trefoil of $\frac{1}{2}$' radius. Fig. 140

Draw an equilateral triangle ABC of side 1" Draw the trefoils with A, B and C as centres and $\frac{1}{2}$ the sides as radii. The outer circles are drawn from the centre of the triangle ABC.



139



140

CHAPTER X.

AREAS AND DIVISION OF AREAS.

104. To draw a square equal in area to a rectangle. Fig. 141.

Let ABCD be a rectangle. Produce AB to F making BF equal to BC. On the whole line AF draw a semi-circle AEF. Produce BC to E meeting the semi-circle in E. Then the square on BE is equal in area to the rectangle ABCD.

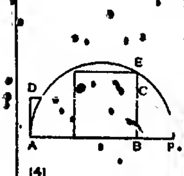
105. To draw a square equal in area to a triangle. Fig. 142

Let ABC be a triangle. From one vertex A, draw a perpendicular AD to the opposite side BC. Bisect AD in E. Complete the rectangle BF on BC with the height DE. Then the rectangle BF is equal in area to the triangle ABC. Then find the side of the square equal in area to this rectangle by problem 104. The square will be equal in area to the triangle.

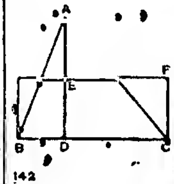
106. To draw a rectangle on a given straight line equal in area to (1) a given rectangle (2) a given square. Figs. 143 and 144.

(1) Let EF be the given straight line and ABCD be the given rectangle.

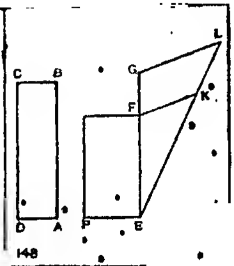
Find the fourth proportional to EF, BC and AB. Produce EF to G making FG = BC. Take a line EK at any angle with EF and make EK equal to AB. Join FK. Produce EK and from G draw GL parallel to FK meeting EK produced in L. Then KL is the fourth proportional *i.e.* EF : FG



141



142



143

: : EK:KL or EF:BC::AB:KL. Draw EP perpendicular to EF and equal to KL. Complete the rectangle PF which is equal to the rectangle ABCD. Fig. 143.

(2) Let ABCD be the given square and EF the given straight line.

Find the third propotional to EF and AB which will be EK the other side of the rectangle on EF. Take EG at any angle with EF and make it equal to AB. Take EM in EF equal to AB. Join FG. From M draw MH parallel to FG. Then EH is the 3rd proportional *ie.*, EF:EG::EM:EH and as EG and EM are both equal to AB ∴ EF:AB::AB:EH.

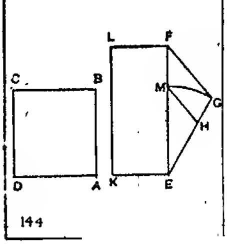
Draw EK perpendicular to EF equal to EH and complete the rectangle KF. Fig 144.

107. To draw a square equal to the difference of the squares on two lines. Fig. 145.

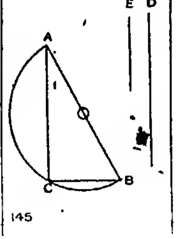
Let D and E be the two given lines. Draw a line AB equal to D and on AB describe a semi-circle. With B as centre and the line E as radius intersect the semi-circle in C. Join AC and BC. Then the square on $AC = AB^2 - BC^2$ *ie.* $D^2 - E^2$.

108. To construct a square equal in area to the sum of three given squares. Fig. 147.

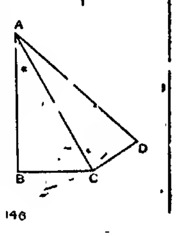
Let AB, BC and CD each be one side of the there given squares. Place AB and BC at right angles at B and join AC. Draw CD perpendicular to AC and join AD. Then the square on AD is equal to $AB^2 + BC^2 + CD^2$.



144



145



146

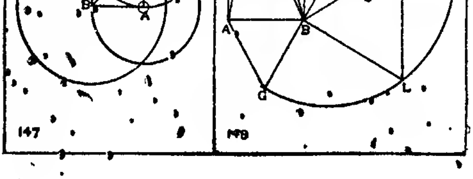
109. To draw a polygon or a circle double of a given similar ploygon or a given cirole. Fig. 147.

Let AB be one side of a given polygon or a radius of a given circle. Draw AC at right angles to AB and equal to it. Join BC. Then a polygon described on BC and similar to the polygon on AB will have its area double of the polygon on AB; or a circle with radius BC will be double of the circle with radius AB

110. To construct an equilateral triangle equal in area to a square or another triangle. Fig. 148.

Let ABCD be the given square or let FAB be the given triangle. Convert the square to a triangle of the same area retaining for it the base AB by producing AD to E making DE equal to AD and joining BE.

On AB one side of the original triangle FAB or the equivalent triangle EAB draw an equilateral triangle AGB. Produce GB one side of the equilateral triangle till it meets a parallel through E or F the vertex opposite to the base AB at H. Draw a semi-circle on the line GH. From B draw BL at right angles to GH to meet the semi-circle in L. On BL construct BKL an equilateral triangle whose area is equal to the square ABCD or the triangle FAB.



147

148

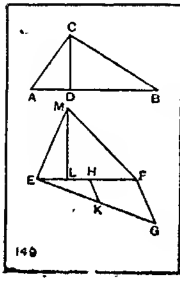
111. On a given base to draw a triangle equal in area to another given triangle. Figs. 149 and 150.

1st method. Fig. 149. Let EF be the given base and ACB be the given triangle. Draw CD the altitude of the given triangle ACB. Find the fourth proportional to the lines EF, AB and CD. EG is drawn equal to AB at any angle to EF. In EF measure EH = CD. Join FG and from H draw HK, parallel to FG. Then EK is the fourth proportional. From L any point in EF draw LM perpendicular to EF and equal to EK. Join ME and MF. Then EMF is the required triangle.

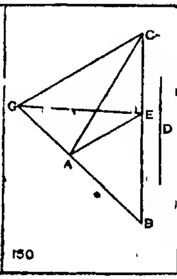
2nd method. Fig. 150. Let ABC be the given triangle and D the given base. On BC set off BE equal to D. Join AE. From C draw CG parallel to AE to meet BA produced in G. Join EG. Then the triangle BEG is equal in area to the triangle ABC and is on the line BE equal to D.

112. To construct a rectangle of a given perimeter and equal in area to a given square. Fig. 151.

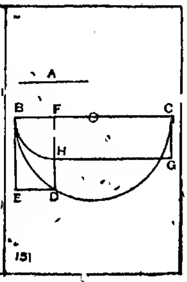
Let A be one side of the square and BC be half of the given perimeter. On BC draw a semi-circle. At B draw BE at right angles to BC and equal to A. Through E draw ED parallel to BC meeting the semi-circle in D. From D draw DF perpendicular to BC. In FD make FH = FB and complete the rectangle FHGC. The rectangle FG is equal to the square on FD = BE = A and its perimeter is double of BC.



149



150



151

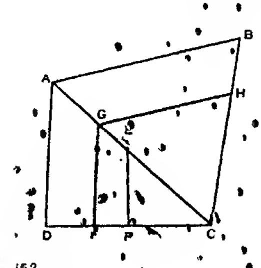
113. To construct a trapezium similar to a given trapezium but having half its area. Fig. 152.

Let ABCD be the given trapezium. Bisect DC one side of the trapezium at P and draw PE perpendicular to it. Make PE = PC or PD. Join CE. With C as centre and CE as radius draw an arc EF meeting CD in F. Join AC a diagonal of the trapezium From F draw FG parallel to AD meeting AC in G. From G draw GH parallel to AD meeting BC in H. Then FCHG is a trapezium similar to DCBA and is half its area.

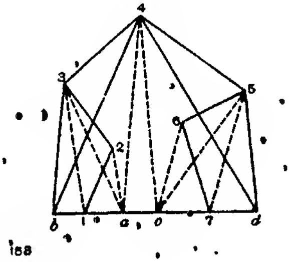
114. To construct a triangle equal in area to any irregular polygon. Fig. 153.

Rule :—Take one side of the polygon as a base or starting line and produce it both ways. Number the corners of the polygon as 1, 2, 3, 4 &c from one of the corners on this line. Join 1 with 3 and draw a line parallel to 13 through 2 to meet the base in *a*. Join 3 with *a*; the polygon is now reduced by one side or angle keeping the same area, Join *a* with corner 4 and draw a line parallel to *a*4 through 3 cutting the base produced in b. Join b with 4. The polygon is now reduced by 2 sides. Similarly reduce the corners of the polygon from the other end of the basee till a triangle is obtained.

For instance take a 7 sided figure as 1,2,3,4,5,6,7. Produce 17 both ways. Join 13 and draw a line parallel to it through 2 meeting 17 in *a*, join *a*4 and draw a line parallel to it through 3 meeting 17 produced in b. Join b4. Commence on the other side. Join 57 and draw a line parallel to it through 6 cutting 17 in c. Take the corners c54. Join c4 and draw a line prallelel to it through 5 cutting 17 produced in d. Join d4. Then by the 4



152



153

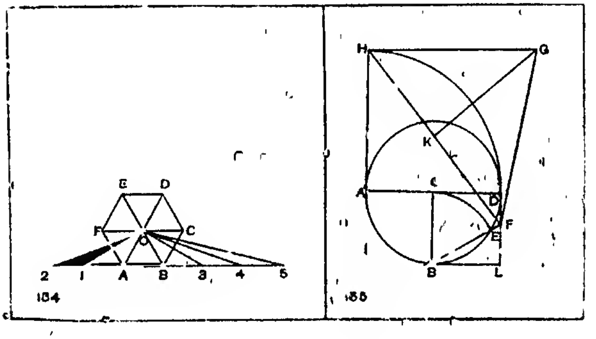
operations 4 sides are reduced of a 7 sided figure and a triangle 4bd is obtained equal in area to the polygon 1234567.

115. To construct a triangle equal in area to any regular polygon. Fig. 154.

Let ABCDEF be a regular polygon for instance a hexagon. Produce one side AB of the polygon both ways. Find O the centre of the polygon and join OA and OB. Make the line obtained after producing AB both ways equal to 6 times AB and mark the six divisions. Join the points with O. In this case 25 is equal to 6 times AB and the triangle O25 is equal in area to the hexagon.

116. To construct a triangle equal in area to a circle. Fig. 155.

Let ABD be a circle. Draw AD a diameter of the circle. Let B be the middle point of the semi-circle ABD. Join B with the centre C of the circle and complete the square BCDL. With B as centre and BC as radius draw an arc cutting the quadrant BD in E. Join BE and produce it to meet DL in F. Draw AH at right angles to AD and equal to it. Join FH. Then FH is equal to the arc of the semi-circle ABD. On FH take any point K and draw KG perpendicular to FH and equal to AD the diameter. Join FG. GH. Then the triangle FGH has area equal to the area of the circle ABD.



117. To construct a square having half the area of a given square. Fig. 156.

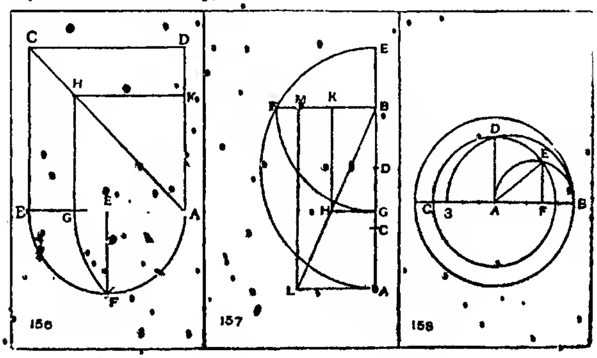
Let ABCD be a given square. Bisect AB in E. On AB draw a semicircle. From E draw EF perpendicular to AB to meet the circumference in F. Take AG in AB equal to AF. Draw the digonal AC of the square. From G draw GH parallel to BC meeting the digonal AC in H. From H draw HK parallel to CD meeting AD in K. Then AGHK is a square $=\frac{1}{2}$ of ABCD.

118. To construct a rectangle $\frac{1}{3}$rd the area of a similar rectangle. Fig. 157.

Let ABML be a given rectangle. Divide AB one side of the rectangle into 3 equal parts in C and D. Produce AB to E making BE equal to BD or one-third of AB. On AE draw a semi-circle AFE. Produce BM to meet the semi-circle in F. Make BG equal to BF and from G draw GH parallel to AL meeting the diagonal BL in H. From H draw HK parallel to LM meeting BM in K. Then BKHG is the required rectangle.

119. To draw a circle $\frac{3}{5}$ths the area of a given circle. Fig. 158.

Let A be the centre of the given circle and AB a radius; divide AB into 5 equal parts. Produce BA to 3 making A3 equal to $\frac{3}{5}$ths of AB. On B3 draw a semi-circle BD3. From A draw AD perpendicular to B3 meeting the semi-circle in D. With centre A and radius equal to AD draw a circle whose area is $\frac{3}{5}$ ths the area of the given circle.



2nd Method. Draw a semi-circle on AB as AEB. From F the third point from A towards AB draw FE perpendicular to AB meeting the semi-circle AEB in E. Join AE; then AE is the radius of the circle which will be $\frac{3}{5}$ths in area of the given circle.

120. To construct a triangle of a given altitude equal in area to another given triangle. Fig. 159.

Let ABC be the given triangle and D the given altitude. From C draw CE perpendicular to AB the base. Make EF equal to D the given altitude. Join FA and draw CG parallel to FA meeting BA produced in G. Join FB and draw CH parallel to FB meeting AB produced in H. Join FG and FH, then FGH is the required triangle.

121. To divide a triangle into any number of equal parts by lines parallel to one of its sides. Fig. 160.

Let ABC be a triangle which is to be divided into 3 equal parts.

Trisect one side AB of the triangle in D and E and draw a semi-circle on AB. From D and E draw perpendiculars DF and EG to AB to meet the semi-circle in F and G. Join BF and BG which are the mean proportionals to BA and BD, and BA and BE respectively. Make BH and BK equal to BF and BG respectively. From H and K draw HL and KM parallel to AC one side of the triangle ABC which will be trisected by HL and KM.

122. To bisect a triangle by a line perpendicular to one side. Fig. 161.

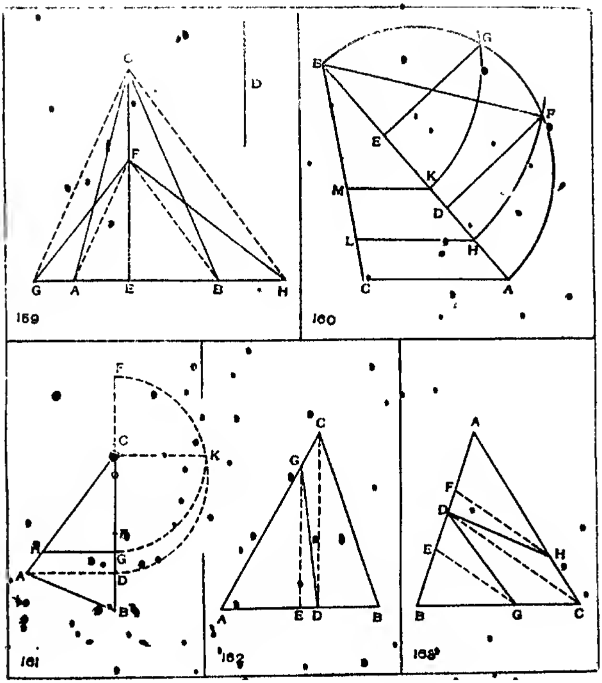
Let ABC be a triangle. Draw AD perpendicular to BC. Bisect BC in E. Produce BC to F making CF equal to CE. Find the mean proportoinal between CD the greater segment of BC and CF half the line. Draw a semi-circle on DF and from C draw CK perpendicular to CF to meet the semi-circle in K. Then CK is the mean proportional. Make CG equal to CK. From G draw GH perpendicular to BC. GH bisects the triangle ABC.

123. To bisect a triangle by a line drawn from a point in one of its sides. Fig. 162.

Let ABC be a triangle and D a point in AB. Bisect AB, (the side in which the point is taken) in E. Join DC and from E draw EG parallel to DC meeting AC in G. Join DG, then DG bisects the triangle ABC.

124. To trisect a triangle by lines drawn from a point in one of the sides. Fig. 163.

Let ABC be a triangle and D a point in AB. Trisect AB in E and F. Join CD. From E and F draw EG and FH parallel to CD meeting the sides of the triangle in G and H respectively. Join DG and DH which will trisect the traingle ABC.



125. To bisect a parallelogram by a line drawn from a given point in one of its sides. Fig. 164.

Let ABCD be a parallelogram and E a point in AB. Draw AC and BD the diagonals of the parallelogram intersecting in O. Join EO and produce it to meet CD in H. Then EH wil blisect the parallelogram.

126. To bisect a quadrilateral figure by a line drawn from one of its angles. Fig. 165.

Let ABCD be a trapezium; it is to be bisected by a line drawn from the corner D. Join the diagonal AC subtended by the corner D. Bisect AC in E. Join ED and EB. The two lines DE, EB bisect the trapezium and if the figure ADEB be reduced to a traingle by a line from D the problem is solved. Join DB the first and the third point of DE, EB and from E the 2nd point draw a line parallel to DB meeting AB in F. Join DF which bisects the quadrilateral.

127. To divide a triangle into any number of equal parts by lines drawn from a point within the triangle. Fig. 166.

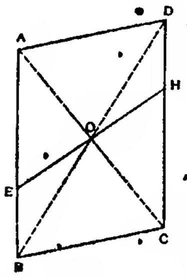
Let ABC be a triangle and D a point in it. The triangle is to be trisected by lines from D. Trisect AC in E and F. Join DE and DF. Draw BG and BH parallel to DE and DF respectively meeting AC in G and H. Join DG, DH and DB. Then these lines trisect the triangle.

128. To divide a circle into any number of equal parts by concentric circles. Say three parts. Fig 167.

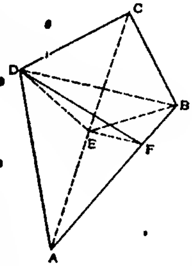
Let AFG be a circle of centre Q. Draw QA a radius. Trisect the radius QA in B and C. On QA draw a semicircle, from the points B and C draw BD and CE perpendiculars to QA meeting the semicircle in D and E. Join QD and QE. With Q as centre and radii equal to QD and QE draw circles which will trisect AGF.

129. To divide a circle into any number of parts equal in area and perimeter. Say three parts. Fig. 168.

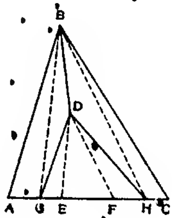
Let ABE be a circle. Draw AB a diameter of the circle ABE. Trisect AB in C and D. On AC draw a semicircle and on CB draw a semicircle on the opposite side. On AD draw a semicircle on the same side as the semicircle on AC and on DB draw a semicircle on the opposite side of it. Now the circle is trisected by the curves AFCGB and AHDKB and it is clear that the perimeters of the three portions are equal in length.



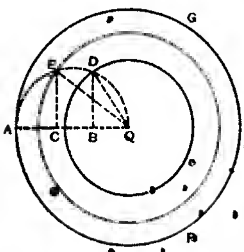
164



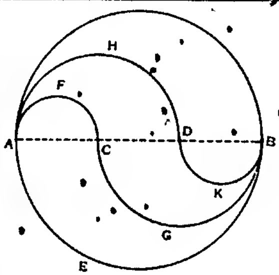
165



166



167



168

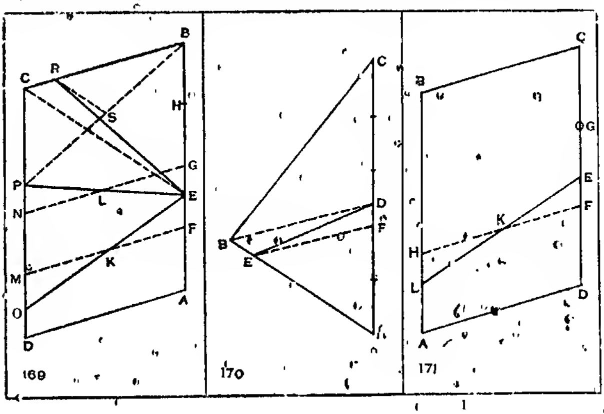
130. To divide a parallelogram into any number of equal parts by lines drawn from a point in one of the sides. Fig. 169.

Let ABCD be a parallelogram and E a point in AB. Let the parallelogram be divided into 4 equal parts. Divide AB in which the point is taken into 4 equal parts in F, G, and H. Through F and G draw FM and GN parallel to AD, by each of which a quarter of the parallelogram is obtained. Bisect FM in K and GN in L. Join EK and EL and produce them to O and P. EO and EP divide the parallelogram each into a fourth part. It is now required to divide the trapezium EPCB into 2 eqnal parts by a line from E. Join PB and bisect it at S. Join EC. Through S draw SR parallel to EC meeting CB in R. Join ER. Then EO, EP and ER are the three required lines.

131. To divide a triangle into two parts, having a given ratio to each other, by a straight line drawn through a given point in one of its sides. Fig. 170.

Let ABC be a traingle and D a point in AC. Divide the line AC in which D is taken in the ratio of 2 : 3 in the point F. Join BD. From F draw FE parallel to BD. Join DE then DE divides the triangle in the ratio 2 : 3.

132. To divide a parallelogram into 2 parts having a given ratio to each other, by a straight line drawn from a given point in one of its sides. Fig. 171.



169 170 171

Let the ratio of the parts be as 1 : 2.

Let ABCD be a parallelogram and E a point in DC one side. Divide DC into 3 equal parts (1+2) at F and G. Through F draw FH parallel to AD. Then FH divides the parallelogram in the ratio 1 : 2. Bisect FH in K and join EK. Produce EK to meet AB in L. Then EL divides the parallelogram in the ratio 1 : 2.

133. To divide a trapezium into 2 equal parts by lines drawn from a given point inside the trapezium. Fig. 172.

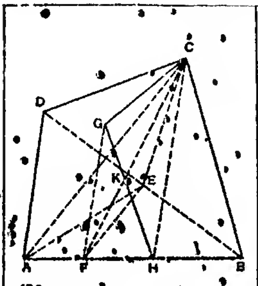
Let ABCD be a trapezium, G a point in it. Join GC. Through the point C draw CF to bisect the field ABCD. (prob. 126 fig. 165). Join GF. Through C draw CH parallel to GF cutting AB in H. Join GH. Then GC and GH bisect the trapezium. Because CF bisects the trapezium and the triangle GCH is equal to the triangle FCH. Take away the common triangle KCH. Then GCK is equal to FKH.

134. To construct a square 3sqr. inches in area.

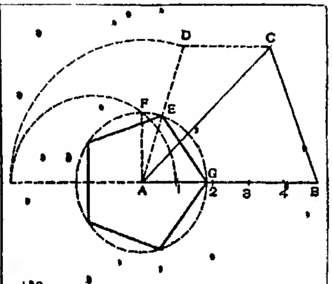
Draw a rectangle of 3 square inches in area. Find the mean proportional between the two adjacent sides of the rectangle which will be a side of the square required (problem 104 Fig. 141)

135 To construct any regular polygon, say a pentagon, equal in area to a given triangle. Fig. 173.

Let CAB be the given triangle. Divide AB into 5 equal parts *i. e.* the same number of parts as the regular polygon will have sides. Draw AD at an angle of 72° with AB *i. e.* at A make an angle equal to the angle at the centre of the required polygon,



172



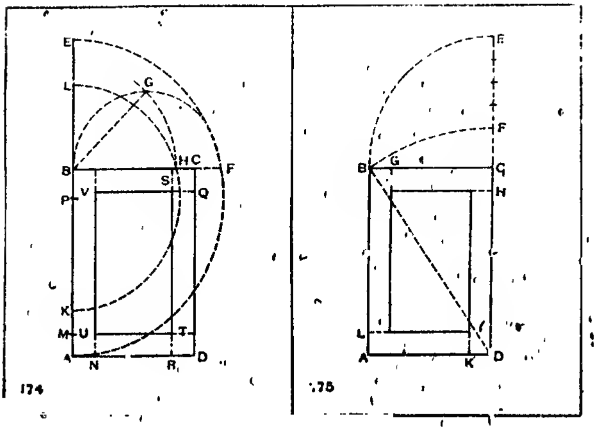
173

in this case a regular pentagon. From C draw CD parallel to AB meeting AD in D. Find the mean proportional AF between AD and AI. With A as centre and AF as radius draw a circle cutting AD in E and AB in G. Join EG which is a side of the regular pentagon to be inscribed in the circle.

139. To draw a rectangle inside another rectangle of half the area of the outer rectangle and leaving spaces of the same width all round. Fig. 174 175.

1st Method., Fig. 174. Let ABCD be a rectangle. Produce AB to E making BE=BC. Find P the middle point of the whole line AE. Find the mean propotional BF between the two adjacent sides of the rectangle AB and BC. The square on BF is equal to the rectangle ABCD. On BF draw a semicircle BGF. Bisect the arc BGF in G. Join BG. Take BH in BF equal to BG. Then the square on BH is half the square on BF. With P as centre and PH as radius draw a semicircle meeting AE in K and L. Then the rectangle contained by BK and BL is equal to the square on BH therefore equal to half the rectangle BA, BE.

Bisect AK in M and set off AN, DR and CQ each equal to AM. Through M,N,R and Q draw lines parallel to the 4 sides of the rectangle, then the rectangle STUV obtained inside the rectangle ABCD is the required figure.



174

175

2nd Method. Fig. 175. Let ABCD be the given rectangle. Join BD. Produce DC and take DF equal to DB and CE equal to CB. Divide FE into 4 equal parts. Then each part is equal to the width to be left round inside the square.

Take BG, CH, DK and AL each equal to F1 or a fourth of FE and through G, H, K and L draw lines parallel to the 4 sides of the given rectangle.

Proof :— Let $DC=a$, $CB=b$

$DE=a+b$. $DF=DB=\sqrt{a^2+b^2}$

Let the width of the path $=x$

Then $(a-2x)(b-2x)=\dfrac{ab}{2}$

$$ab-2bx-2ax+4x^2=\frac{ab}{2}$$

$$4x^2-2x(a+b)+ab=\frac{ab}{2}$$

$$4x^2-2x(a+b)+\frac{ab}{2}=0$$

$$x=\frac{(a+b)\pm\sqrt{4(a+b)^2-\frac{16ab}{2}}}{8}$$

$$=\frac{2(a+b)\pm\sqrt{4(a^2+2ab+b^2)-8ab}}{8}$$

$$=\frac{2(a+b)\pm\sqrt{4a^2+4b^2}}{8}$$

$$=\frac{2(a+b)\pm\sqrt{4(a^2+b^2)}}{8}=\frac{2(a+b)\pm2\sqrt{a^2+b^2}}{8}$$

$$=2\left\{\frac{a+b-\sqrt{(a^2+b^2)}}{8}\right\}=\frac{a+b-\sqrt{a^2+b^2}}{4}$$

i. e. $FE=4x=a+b-\sqrt{a^2+b^2}=DE-DB$ *i. e.* DF.

EXERCISES.

CHAPTER IV.

1. Draw a line 3″ long. From a point below the line draw a perpendicular. Trisect the right angle thus formed.

2. Divide a line 5″ long into six equal parts. Draw parallel lines half an inch apart, through these divisions.

3. Set off an angle of $22\frac{1}{2}°$ and 75° without using protractor.

4. Draw two parallel lines A and B $1\frac{1}{4}$″ apart. Take a point P $\frac{1}{2}$″ above A. Through P draw a line cutting the given lines in such a way that the portion intercepted between the lines will be $1\frac{3}{8}$″.

5. Bisect a given line by the use of two set squares with angles of 45° and 60°.

6. Draw lines of the following lengths $2\frac{1}{4}$″, 1·25″, 3·5″, $1\frac{7}{8}$″, 3″.

7. Draw two parallel lines $2\frac{7}{8}$″ long and $1\frac{3}{8}$″ apart.

8. Find a point C in a line AB ($2\frac{1}{2}$ inches long) produced, so that AC : AB as 5 : 4.

9. Draw two lines meeting at an angle of $67\frac{1}{2}°$ and between them place a line $2\frac{1}{2}$ inches long making 60° with one of them.

10. Find a line which shall have the same ratio to a line $2\frac{1}{5}$″ long that 5 has to 3.

11. From the left extremity of a given line obtain $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$th of it.

12. Divide a line 3·75″ long into 3 parts A, B and C so that B is double of A and C $1\frac{1}{2}$ times B.

EXERCISES.

CHAPTER V.

1. Draw a right angled triangle with one angle 30° and hypotenuse $2\frac{1}{2}$″.

2. On a base $1\frac{7}{8}$″ long construct an isosceles triangle with a vertical angle 30°.

3. On a base $2\frac{1}{8}$″ long draw a segment of a circle containing an angle of 120°.

4. Make a triangle sides 2·5″, 1·25″ and 1·75″ long.

5. Draw a triangle vertical angle 30° base 1·7″ and sides as 4 : 5.

6. Construct a rhombus with sides $1\frac{5}{8}$″ long and one of its angles 60°.

7. Make a square to contain 5·36 sqr. inches.

8. Describe the figure ABCD when AD = 1″, AC = $1\frac{1}{2}$″, angle DAC = 60°, BC = $\frac{1}{2}$AC, AB = $\frac{1}{3}$BC.

9. Construct a rhomboid, adjacent sides 3″ and 2″ and diagonal 4″.

10. Draw a triangle, having given (1) two sides and an angle opposite to one, (2) two angles and the intermediate side, (3) two angles and a side opposite to one of them.

11. Construct a triangle having the base $1\frac{1}{2}$″, altitude = 1″ and perimeter $3\frac{3}{4}$″.

12. The three medians of a triangle are $\frac{7}{8}$″, $1\frac{1}{8}$″ and $1\frac{1}{2}$″, draw the triangle.

13. Construct a triangle having its base $=2''$, altitude $=2\frac{1}{4}''$, and ratio of sides 4 : 7.

14. Draw a right angle and trisect it : on the same figure construct the following angles *viz.*, $7\frac{1}{2}°$, $15°$, $22°\frac{1}{2}$, $30°$, $37°\frac{1}{2}$, $45°$, $60°$, $67\frac{1}{2}°$, $75°$.

15 Construct an isosceles triangle, altitude $AB=2\frac{1}{4}''$ making angles of $25°$ with the equal sides of the triangle.

16. Construct a rhombus one side is equal to $1\frac{1}{4}$ inches and the diagonal equal to 2 inches.

17. Construct an equilateral triangle 2 inches high.

18. Construct a right angled triangle having angles in the proportion of 3 : 4 : 6

19. Take three points A,B,C in a straight line, $AB=1\frac{1}{2}$ inches $BC=2\frac{1}{4}$ in. B between A and C. Draw a rectangle, the sides of which are in the ratio 2 : 3 vertex at B, the two sides passing respectively through A and C Hint :—Bisect AB and BC and through the points of bisection draw lines at right angles to AB and BC and equal to halves of the line respectively Join the ends of these perpendiculars with B which will form the sides of the rectangle required.

20. The diagonals of a rhomboid $2\cdot3''$ and $3\cdot2''$ long contain an angle of $60°$. Construct the rhomboid.

21. Construct a square $3''$ sides, bisect the sides, and join the adjacent points of bisection, thus obtaining a second square ; bisect the sides of this square and obtain a third square. Continue the process until 4 squares have been drawn. Measure the diagonal of the smallest square.

EXERCISES.

CHAPTER VI.

1. In a circle $1\frac{3}{8}''$ radius inscribe a regular heptagon. Find the length of one side and the value of one interior angle of the polygon.

2. How many degrees are there in each of the angles at the centre of a nonagon.

3. Construct a regular polygon with one side equal to $1''$ in length and one angle equal to $140°$.

4. Construct a regular polygon on the chord of an arc of $72°$.

5. Construct a regular pentagon, diagonal $3''$.

6. Construct a regular polygon one side $1\frac{1}{2}''$ and an angle $120°$.

7. Construct an irregular pentagon ABCDE from the following data : sides $AB=2\frac{1}{4}''$, $BC=1\frac{3}{4}''$, $CD=2''$ $DE=1\frac{1}{4}''$ diagonal $AD=3''$ angles $ABC=120°$, $CDE=112\cdot5°$.

8 The sides of a quadrilateral ABCD are as follows $AB=1''$, $BC=1''$, $CD=1\frac{1}{4}''$, $AD=1\frac{1}{2}''$, diagonal* $CD=1\frac{1}{2}''$ construct it and draw a similar figure of perimeter $4''$.

9. O is a point within a quadrilateral figure ABCD. Construct the figure from the following dimensions :—

Angles : $AOB=115°$, $BOC=55°$, $AOD=85°$

Lengths : $OA=1\frac{1}{4}''$, $OB=1\frac{3}{8}''$, $BC=1\frac{3}{4}''$, $BD=3''$.

10. Draw a quadrilateral figure ABCD with the following dimensions :—

$AB=2''$, $BC=1\frac{1}{2}''$, $AD=1\frac{1}{2}''$. The diagonal* $BD=2''$, the diagonal $AC=2\frac{1}{2}''$.

* See Definition 12 Chap. III.

EXERCISES.

CHAPTER VII.

1. Within a given regular polygon to inscribe another similar figure, having its sides parallel to and equidistant from those of the given figure, the length of one side being given.

2. About a givon regular polygon to describs another similar figure having its sides parallel to and equidistant from those of the given figure, the length of one side being givon.

3. In a square 2″ sides inscribe another having a side of 1.75″.

4. Construct a square of 2½″ sides and in it inscribe an isosceles triangle with 1¾″ base; inscribe within the triangle a rectangle, one side of which is 1¼″.

5. Construct a quadrilateral base 3″, base angles 90° and 75°, sides 2″ and 2″. Within it inscribe a parallelogram having a side of 2″.

6. Within a square of 3″ sides inscribe the largest possible equilateral triangle.

7. Within a square of 2½″ side inscribe an octagon, so that the alternate sides of the octagon shall coincide with the sides of the square.

8. In a triangle ABC inscribs an equilateral triangle with one vertex bisecting AB.

9. Construct within a given triangle and equidistant from the sides of it, a similar triangle, the base of which is equal to a given line.

10. The same but the triangle is to be drawn outside the given triangle.

EXERCISES.

CHAPTER VIII.

1. Plot without the aid of protractor the following angles, 15° and 75°.

2. Plot the following angles by protractor and verify them by a scale of chords of 3″ radius. 27°; 49°; 82°, 123°; 163° and 295°.

3. Construct a scale of chords on a radius 3·75″ to read 5°. By means of this scale plot an angle of 65°.

4. By means of a scale of chords of 3″ radius construct a triangle base 4″ angles at the base 80° and 20°. Measure the 2 sides, correct to two places of decimals.

5. Without the aid of a protractor construct an isosceles triangle on a line 3½ inches long with angle at base=75° and construct a square equal to it in area.

6. Three posts B, C, D are in a straight line at intervals of 100 yards. An observer at A finds that the angle BAC is 20° and CAD =30°. Obtain the position of A. One inch represents 200 feet.

EXERCISES.

CHAPTER IX.

1. Draw a tangent to a point in the arc of a given circle without taking help of the centre.

2 Mark three points not in a straight line. Find a point equidistant from them.

3. Two circles radii $1\frac{1}{2}''$ and $1''$ have their centres $2''$ apart. Draw the common exterior and interior tangents.

4. Draw two circles of $1''$ and $\frac{1}{2}''$ radius with their centres $2\frac{1}{2}''$ apart; draw another circle tangential to both externally.

5. Draw two lines at an angle of 30° and a third line cutting them both at any convenient angle; draw two circles tangential to all the three lines.

6. In a circle of $1''$ diameter, inscribe a quadrifoil having tangential arcs.

7. Construct a pentagon of $\frac{3}{4}''$ sides and about it describe a cinquefoil having adjacent diameters (cinquefoil is of 5 semi-circles).

8. Construct a square with sides of $2\cdot3''$, and inscribe four equal circles within it; each circle to touch two others as well as one side of the square.

9. Within a triangle of $2\cdot3''$ sides, inscribe six equal circles.

10. Within a circle of $1\cdot5''$ radius inscribe 5 equal circles.

11. To inscribe a circle which shall have its centre on a given line CD, and shall touch a given line AB and a given circle.

Hint :—Draw any line EF at right angles to AB and any line through the centre O of the given circle as OG towards EF cutting the circumference in H. Along EF set off any lengths E1, E2, E 3&c. and along HG set off H1, H2 and H3 &c respectively equal to these. With O as centre describe arcs through the points 1, 2, 3, &c. on HG to meet lines drawn parallel to AB through the corresponding points 1, 2, 3 &c. on EF. These arcs and corresponding lines intersect at points a, b, c &c. through which draw a curve. This curve is the locus of the centre of a circle which moves so as always to touch the given line and circle and the point in which the curve intersects the line CD is the centre of the required circle whose radius is equal to the perpendicular from the point to the line AB.

12. A point P is 2·75 inches from the centre of a circle of 1 inch radius. From P draw a line to cut the circumference of the circle in two points A and B so that PA : AB : : 2 : 3.

13. Find a third point C on the circumference of a circle such that CA : CB : : 5 : 3. A and B are two other points on the circumference.

14. In a circle, radius $2''$ describe 3 equal circles each touching two others and the containing circle.

EXERCISES.

CHAPTER X.

1. Construct a parallelogram on a line 3″ long equal to a square side $2\frac{1}{4}''$.

2. Construct a triangle, perimeter 5″, sides as 5 : 4 : 3 and make an equilateral triangle equal to it.

3. Divide a triangle into 3 parts in the ratio 1 : 2 : 3 by lines drawn from one of its angles.

4. Construct a right angled triangle base 2″ and area 2·58 square inches.

5. Construct a triangle area 3 square in. with sides as 3 : 4 : 5.

6. Construct an equilateral triangle equal in area to the difference between two other equilateral triangles with sides of 1·3″ and 2·25″ respectively.

7. Draw a regular heptagon on a side of 1·25″ ; and construct a similar polygon $\frac{3}{4}$ths of its area.

8. Construct a trapezium with sides of $1\frac{1}{4}''$ $2\frac{3}{8}''$, $3\frac{1}{4}''$ and $2\frac{1}{4}''$, one of its angles to be 60°. Bisect it through one of its angles.

9. Draw a pentagon on a 1·5″ side ; and construct a rectangle equal to it on a 3″ side.

10. Construct any irregular octagon and divide it into seven equal parts.

11. Two triangles ABC, DEF are given It is required to draw a triangle def, with its vertices d, e, f, in BC, CA, AB and its sides de, ef, and fd, parallel to DE, EF and FD.

12. A triangle ABC and a quadrilateral DEFG are given. It is required to draw a quadrilateral defg similar to DEFG with its side de in AB, and its vertices f and g in BC and CA respectively.

13. In ABC inscribe an equilateral triangle with one vertex bisecting AB.

14. Draw a square ABCD of 2 inches side and through C draw a line meeting AB in P and DA produced in Q so that the area of the triangle PAQ shall be double of the triangle PBC.

15. Draw a circle $\frac{3}{4}$ths the area of a given circle, and divide it by concentric circles into 3 equal parts.

16. Construct an isosceles triangle with an area of 3 square inches and having a vertical angle of 30°.

17. Divide a square of 2 inches side into 3 equal areas by lines parallel to a diagonal.

18. Within a circle of $1\frac{1}{2}$ inches radius inscribe a rectangle with an area of 2 square inches.

19. Within a square of 2 inches side inscribe a square having its angles in the sides of the first, and its area to the area of the first square as 2 : 3.

20. Describe a square equal to the difference of two squares whose sides are 2·75″ and 1·45″.

28. A square has its diagonal 0·23″ longer than its side. Construct it.

CHAPTER XI.

PLAIN SCALES, DIAGONAL SCALES AND COMPARATIVE SCALES.

When an object is so large that it cannot be represented on paper full size, its drawing is done by reducing each line in the drawing to a fixed and known proportion to the line it represents. This proportion or ratio of reduction is known as the scale of the drawing and when it is expressed in fraction it is called the Representative Fraction of the scale or more commonly the R. F. of the scale. Suppose on a drawing of a culvert the scale is written as 4 ft $= 1''$. From this it is to be inferred that every inch on the drawing represents 4 ft. or 48 inches on the culvert and the ratio of reduction is $\frac{1}{48}$ which is the R. F. of the scale. Usually the scale is expressed by the length represented by 1 inch in case of Engineering or Mechanical drawings as $2' = 1''$ or $4' = 1''$; and in case of topographical drawings either by the length represented by 1 inch or by the number of inches representing a length of one mile as when R.F. $= \frac{1}{3960}$, the scale is either $330' = 1''$ or 1 mile $= 16''$.

For the convenience of measuring off a distance from the drawing in order to know the real length of the line it represents, a graduated straight line is attached to all drawings called the "scale" in addition to the written representation of the scale.

Scales may be drawn to show two units of measure as feet and inches, tens of yards and single yards or miles and furlongs.

&c. These are called plain scales When three units of measure have to be shown as yards, feet, and inches: or hundreds of feet, tens of feet and a single foot; or a foot, tenths of a foot and hundredths of a foot, either a *diagonal* or a *vernier* scale is to be constructed by which very minute divisions can be attained which is impracticable by the plain method of division.

Comparative scales:—It is sometimes necessary, when the scale of a drawing is adapted to one unit of length suitable for one place, to construct another with the same representative fraction but having a different unit of length prevalent in another place. These scales are known as comparative scales They are graduated differently with different units though the R F. in the two cases is the same. They are plain scales either drawn separately or one over the other. They are required for maps of countries which have different standards of measuring distances.

Scales will be treated in the following order:—

1. Plain scales, 2. Diagonal scales, 3. Comparative scales, 4. Vernier scales.

Plain scales:—In all scales it is evident that if they fulfil the functions explained above, any length on the scale must bear the proportion expressed by the representative fraction of the scale to the real length it represents.

Scales are usually made about 6 or 7 inches long, but it is sometimes made smaller or longer when it is convenient to do so.

A convenient rule to draw scales is to assume a certain length to be represented by the scale, which will occupy about 6 inches of space on paper. The number to be assumed should usually, though not necessarily, if it be more than 9, be either 10 or a multiple of 10 as 50, 80 or 100. Draw a straight line and measure on it the number of inches representing the length

assumed. Divide the line into as many units or tens of units as the assumed number expresses. These are called the primary divisions of the scale. To show these divisions or spaces more clearly, another line is drawn parallel to the first and about $\frac{1}{10}$th of an inch below it. Draw vertical lines through these divisions from the 2nd line to about $\frac{1}{10}$th of an inch above the first horizontal line. The last but one point from the left is always marked zero. From this point mark the primary divisions to the right as 10, 20, 30, &c or 100, 200, 300 &c. as the case may be. The division on the left of the zero point is to be subdivided, either into 10 equal parts if the primary divisions represent 10, or a multiple of 10, or into an aliquot part representing a sub-division of the primary division, expressed by a unit of the linear measurement, for instance into 12 parts for inches in case of a foot, or into 8 parts for furlongs in case of a single mile of primary divisions. The sub-divisions are to be marked from the right i. e. from the zero point of the scale to the left. The zero point connects the two divisions in such a way, that by one stretch of the legs of the compasses, a measurement may be taken which comprises both the primary and the sub-divisions of the scale.

A few examples are given below to explain the construction of scales. It will be seen, that the principle is the same in all cases.

1. Construct a scale of 1′=1″ Fig. 176

Let the length of the scale show 6 feet which will be represented by a line 6 inches long. Divide it into 6 equal parts to show a single foot of primary divisions. Divide the first division on the left into 12 equal parts to represent inches.

2 To construct a scale of feet and inches R. F.$=\frac{1}{30}$.

Fig. 117. Assume the length of the scale to show 10 feet which will be represented by $\frac{10}{2\frac{1}{2}}=4$ inches.

"Take a line 4 inches long. It represents 10 feet. Divide it into 10 equal parts." Then each part of the primary division represents 1 foot. Divide the first division into 12 equal parts which will show inches.

3. To construct a scale of 12 yards=1". Fig. 178.

Assume the length of the scale to show 70 yards The number 70 is arrived at by multiplying $12 \times 6 = 72$ and its nearest multiple of 10 is 70.

70 yards will be represented by $\frac{70}{12} = 5.83$ inches. Take a straight line and measure on it a distance of 5.83 inches, from the decimal diagonal scale on the back of a rectangular protractor, the construction of which will be explained further on.

This length is 70 yards. Divide it into 7 equal parts and subdivide the 1st left hand division into 10 equal parts to show a single yard.

4. To construct a scale of 6"=1 mile showing chains of 100 feet. Fig. 179.

$\therefore$ 1 mile = 5280 feet = 6"

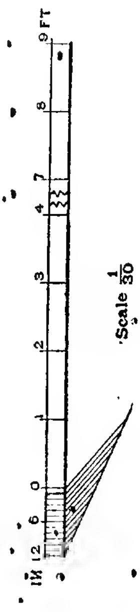
We can assume the length of the scale to be 5000 feet.

Then 5000 ft. will be represented by $\frac{5000 \times 6}{5280} = 5{\cdot}68$ inches.

Divide the length of 5·68 inches which represents 5000 feet or 50 chains into 5 equal parts; each portion of the primary division will be 10 chains. Divide the 1st division on the left into 10 equal parts to show a chain or 100 feet.

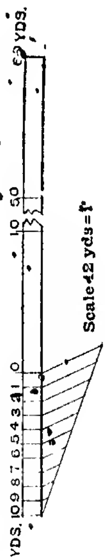
Scale 1′ = 1

176



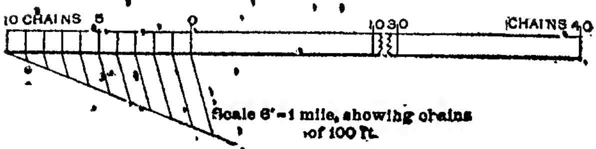
Scale $\frac{1}{30}$

177



Scale 12 yds = 1″

178



Scale 6″ = 1 mile, showing chains of 100 ft.

179

5. To construct a scale of 8 inches=1 mile showing 10 paces. A pace=30 inches. Fig. 180.

A pace = 30 inches = $2\frac{1}{2}$ feet.

∴ 10 paces = 25 feet.

5280 ft. = 8 inches. Assume the length of the scale to show 1000, paces i. e. 2500 feet which will be represented by $\frac{2500 \times 8}{5280} = 3{\cdot}78$ inches.

Divide the length 3·78 inches into 10 equal parts; each part will represent 100 paces. Divide the first primary division into 10 equal parts, then spaces of 10 paces will be shown.

6 A map is 36 inches long and 24 inches broad; it represents an area of 20 acres. Draw the scale of the map to show poles and yards. 4840 sqr. yds = 1 acre Fig. 181.

36 × 24 = 864 sqr. inches represent 20 acres.

20 acres = 20 × 4840 sqr. yds. = 96800 sqr. yds.

864 sqr. inches represent 96800 sqr. yds. Divide by 8

108 sqr. inches ... 12100 sqr. yds.

$6\sqrt{3}$ inches ... 110 yds.

5.19 inches ... 55 yds.

The scale is to show poles and yards. Assume the length of the scale to be 10 poles or 55 yds.

10 poles will be represented by 5.19 inches.

Divide the length 5.19 inches into 10 equal parts each part is a pole. A pole = $5\frac{1}{2}$ yds. Therefore divide 2 poles or 11 yds. into 11 equal parts to show 1 yard. The zero point in this case is the 2nd point from the left

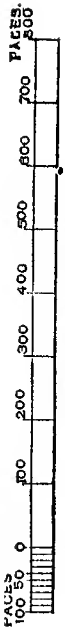
7. A scale of $\frac{1}{15000}$ to take off intervals of time, adapted to the trot of a horse. A horse goes over 250 yards per minute at a fast trot. Show 10 minutes. Fig 182.

R. F. = $\frac{1}{15000}$ or 1250 ft. = 1″

The horse goes over 250 yds in 1 minute therefore in 10 minutes the horse goes 2500 yds = 7500 ft; which will be represented by $\frac{7500}{1250} = 6''$

Take a line 6 inches long. It represents 10 minutes. Divide it into 10 equal parts each portion is a minute. Divide the first space into 6 equal parts each portion will show 10 seconds

Diagonal scales:—If we want a fractional portion of a secondary division which is too small to be divided we can obtain it by adopting the following method:



Scale 1 mile = 8 Inches
Showing 10 paces, a pace = 2½ feet
180

Scale 55 yds. = 5·19 Inches
or 10 Poles = 5·19 Showing
Poles and yards
181

Scale $\frac{1}{15000}$ adapted to 250 yards
per minute.
182

183

Let AB be a small length of a subdivision of a scale. Fig. 183. It is to be divided into 10 equal parts. Draw AC perpendicular to AB and set off 10 equal spaces of any convenient length as A1, 12, 23 &c to 9C. Join CB and from the points of division in AC draw lines parallel to AB. Then the small space at division 9 is $\frac{1}{10}$th of AB. The parallel through 8 is $\frac{2}{10}$ ths, through 7 is $\frac{3}{10}$ ths, of AB and so on.

8. Construct a decimal diagonal scale of inches showing tenths and hundredths Fig. 184.

Take a line 6 inches long and divide it into 6 equal parts each part is 1 inch. Divide the first division into 10 equal parts; each part is $\frac{1}{10}$th of an inch. To show hundredths of an inch adopt the diagonal method shown above. Draw vertical lines through each inch division. Set off on the 1st vertical line 10 equal spaces. The horizontal lines are to be drawn from the points in the first vertical line to the last. The zero point of a diagonal scale is always placed where the first diagonal meets the 2nd vertical line from the left.

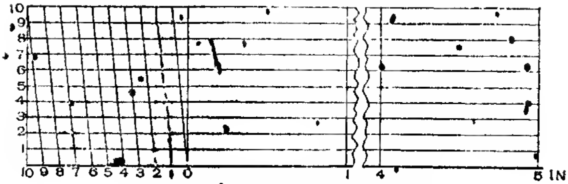
When the number of the sub divisions of the scale is 10 and 10 parallel lines are drawn for the diagonal method the scale is called a decimal diagonal scale or else it is a diagonal scale.

9. To Construct a diagonal scale showing miles, furlongs and gunters chains, the scale is 1 mile =1″. Fig. 185.

Take a line 6 inches long to show 6 miles. Divide it into 6 equal parts for a mile. Divide the first division into 8 equal parts to show a furlong. A furlong is 660 feet or 10 chains. Draw 10 equidistant parallel lines and draw the diagonals. A distance of 2 miles, 3 furlongs and 7 chains can be shown by arrow heads.

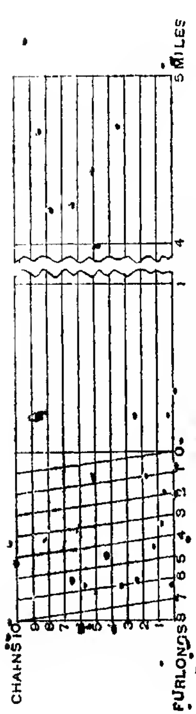
10. A diagonal scale of 8 ft = 1″ showing inches diagonally. Fig. 186.

Assume the length of the scale to show 50 feet. Then 50 feet will be represented by $\frac{50}{8}=6{\cdot}25$ inches. Take a line 6·25 inches long and divide it into 5 equal parts; each portion will show 10 feet. Divide the first division into 10 equal parts for a single foot, and draw 12 equidistant parallel lines for getting inches by the diagonal method.



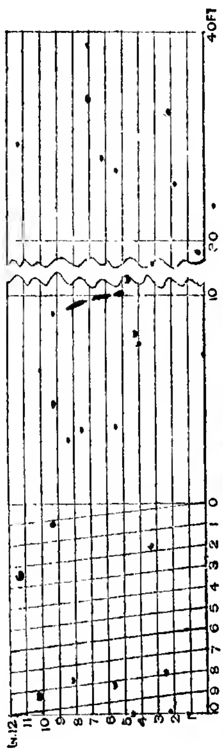
Decimal Diagonal Scale of Inches.

184



Diagonal Scale of 1 mile = 1'
showing Miles, Furlong and Chains

185



Scale 8' = 1' showing inches

186

11. A scale of bighas and cottahs corresponding to 330 ft. = 1". Show a cottah by the diagonal method. A bigha = 120ft = 20 cottahs. Fig. 187.

A bigha = 120 feet therefore 1 inch will represent little less than 3 bighas. We can assume the length of the scale to show 10 bighas i e. 1200 feet, which will be represented by $\frac{1200}{330}$ = 3·63 inches.

Here a line 3·63 inches long is taken which is 10 bighas. Divide it into 10 equal parts to show a bigha and draw vertical lines through the points. The diagonal method of division is adopted here to obtain a subdivision of the primary division as a cottah is $\frac{1}{20}$th of a bigha. The space of a bigha can easily be divided into 4 equal parts which will give 5 cottah spaces and 4 equidistant parallel lines can be drawn to get a fifth of it A length of 5 bighas and 13 cottahs can be marked by arrow heads

Comparative scales, examples :—

12. To construct a scale of English miles comparative to a scale of Russian Versts, 30 versts = 1". 1 verst = 1166·68 yards Fig. 188.

$\frac{30 \times 1166 \cdot 68}{1760}$ miles = 1" i e. 19·88 miles = 1".

Assume the length of the scale to show 100 miles.

$\frac{100}{19 \cdot 88}$ = 5·03 inches which represents 100 miles.

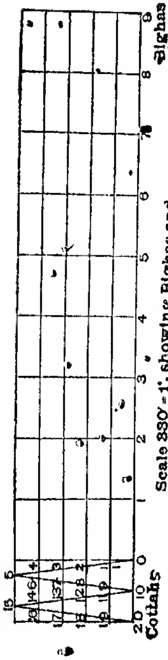
Take a line 5·03 inches long and divide it into 10 equal parts; each portion will represent 10 miles. Subdivide the 1st division into ten equal parts each portion is a mile. 100 versts will be represented by $\frac{100}{30}$ = 3·33 inches. Take a length 3·33 inches long on the lower side of the mile scale and divide it into 10 equal parts; each portion is 10 verts; Divide the left hand one into 10 equal parts each part is a verst.

13 To construct a scale of Bengali "Half Koshes" subdivided into fourths comparative to a scale of 2 miles = 1". Fig. 189.

A kosh = 3600 yards.

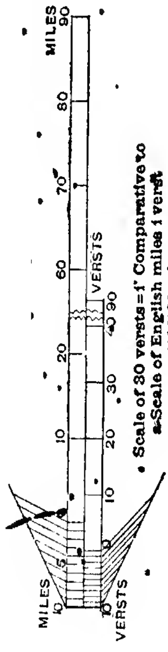
A kosh is little over two miles. We can assume the length of the scale to show 5 koshes or 10 half koshes.

5 koshes will be represented by $\frac{5 \times 3600}{1760 \times 2}$ = 5·11 inches.



Scale 330' = 1', showing Bighas and diagonally Cottahs A Bigha = 120'

187



Scale of 30 versts = 1' Comparative to a Scale of English miles 1 verst = 1166·66 yds.

188

Scale of 2 Miles = 1' Comparative to a Scale of Bengali Koshes A Kosh = 3600 yds.

189

Take a line 5·11 inches long. It is 10 half koshes in length. Divide it into 10 equal parts each portion is a half kosh. Divide the left hand one into 4 equal parts each portion is a fourth or a half poah.

On the lower side of the scale take a length 5 inches long and divide it into 10 equal parts each portion is a mile. Divide the left hand one into 8 equal parts each part is a furlong.

14. To construct a scale of French metres comparative to an English scale of 80 yds=1". Fig. 190.

1 metre = 1·0936 yards.

Assume the length of the scale to show 400 metres.

$$\frac{400 \times 1 \cdot 0936}{80} = 5 \cdot 47 \text{ inches will represent 400 metres.}$$

Take a line 5·47 inches long and divide it into 4 equal parts each part is 100 metres. Divide the left one into 10 equal parts each part is a 10 metre space.

On the lower side of the scale take a length 5 inches long which will represent 400 yards. Divide it into 4 equal parts each part is 100 yards.

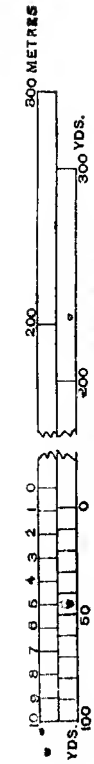
Vernier scales :—

Vernier scales are sometimes used instead of diagonal scales to get a very minute division or a division of a sub-division. The principle of its construction is as follows :—

If a length representing n units of measurement be divided into n equal parts each part will be a unit. Now, if a line equal to $n+1$ of these units be taken and divided into n equal parts each part will be $\frac{n+1}{n}$ or $1\frac{1}{n}$ units. The difference between one sub-division of the last and one sub-division of the former is $\frac{n+1}{n}-\frac{n}{n}=\frac{1}{n}$ of the original unit of sub-division.

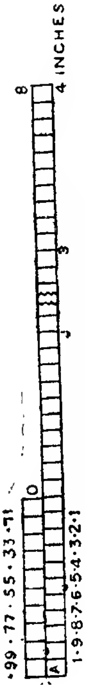
Similarly the difference between the two sub-divisions is $\frac{2}{n}$, and so on.

The vernier Scale can be either straight for liner measurements or circular for angular measurements. The circular vernier is adapted for the sub-division of a degree or a half degree on the limb of a theodolite. This vernier does not fall within the scope of geometrical drawings and is therefore omitted.



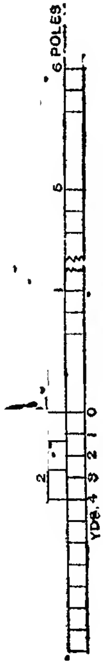
Scale 80 yds.=1" Comparative to a Scale of French metres 1 metre=1·0936 yds.

190



Scale of Inches showing 10ths and 100ths by Vernier

191



Scale $\frac{1}{276}$ to show poles and yards and by Vernier feet

192

There is one peculiarity in the construction of the Vernier Scales, that is, all the primary divisions are to be subdivided and not the one on the left. The reason will be seen from the examples.

15 To construct a vernier scale to read 10ths and 100ths of an inch and mark on it a length of 2 76 inches. Fig. 191.

Take a line 5 inches long and divide it into 5 equal parts to show inches. Divide each inch length into 10 equal parts for the $\frac{1}{10}$th of an inch. Draw the upper line of the scale and the vertical divisions. Produce the upper line to the left and commencing from the point of the 10th subdivision measure a length on the left equal to 11 of these subdivisions and divide it into 10 equal parts Draw a third parallel line over this portion on the left which is called the vernier and draw the vertical divisions of the vernier. Put 0 on the 10th place or the commencement of the vernier and mark below every 10th place from there as 1, 2, 3, 4 inches on the right. Mark the divisions on the left as 1, 2, 3 up to 10 on the lower side and on the upper side 11, 22, 33 &c to 110. The lower division are ·1, ·2 3 &c. and the upper numbers are ·11, ·22, ·33 &c.

Each of the vernier division is 11 in. in length. To measure 2·76 we can have 66 in the vernier and 2·1 in the primary scale. It is to be shown by arrow heads on the scale. To measure 0·83 we can have ·33 in the vernier and ·5 in the primary scale on the right of the zero point To measure ·37 we can have ·7 on the vernier and from it we can substract ·4 from the divisions below the vernier on the left of the zero

16. To construct a scale of $\frac{1}{276}$ to show poles and yards and by a vernier to read feet. Fig. 192.

R. F. $= \frac{1}{276}$ i. e. the scale is 23ft $= 1''$.

Assume the length of the scale to be 8 poles. 8 poles will be represented by $\dfrac{8 \times 5\frac{1}{2} \times 3}{23} = 5{\cdot}74$ inches

Take a line 5·74 inches long and divide it into 8 equal parts. Each part is a pole. Divide the first two divisions into 11 equal parts. Then each part is a yard. Take 4 of these in the vernier and divide it into 3 equal parts. Each part in the vernier will be 1 yd. 1 ft.

EXERCISES CHAPTER XI.

SCALES,—PLAIN, DIAGONAL, COMPARATIVE AND VERNIER.

1. Construct a scale to measure yards and feet. The R. F. $\frac{1}{[illegible]}$.

2. Construct a scale of metres R. F. = $\frac{1}{[illegible]}$. 1 metre = 1·0936 yds.

3. On a map, the distance between two places known to be 20 mile apart measures 8". Construct a diagonal scale to measure miles and furlongs.

4. Construct a scale of $\frac{1}{63360}$.

5. The scale of an Indian plan is drawn in Haths. 1 inch represents 6·75 haths. It is required to draw a comparative scale of feet. 1 Hath = 18 inches.

6. On a map showing a scale of kilometres 48 are found to equal 3". What is the R F. Construct a comparative scale of English miles. 1 kilometre = 1094 yds.

7. A diagonal scale of $1\frac{1}{2}$ inches = 1ft., Show feet, inches and eighths of an inch diagonally.

8. A scale of 8 inches to 1 mile to read to 20 paces and by vernier to 5 paces, 1 pace = 30 inches.

9. Construct a diagonal scale of $1\frac{1}{2}$ inches to the mile to show miles, furlongs and chains and mark on it a length of 2 miles, 3 furlongs and 2 chains. What is the R. F. of the scale.

10. A scale of $\frac{1}{105}$ to show feet and by vernier inches.

11. Make a scale of knots comparative to a scale of 8 miles = 1 inch. A knot = 1·15 miles.

12. Construct a scale of 1 mile = 3 inches showing diagonally spaces 10 yards.

13. A distanceof 11 miles 3 furlongs is shown on a map by $4\frac{1}{2}$ inches. Draw a scale for the map showing furlongs by diagonal division.

14. Construct a scale of $\frac{3}{4}"$ = 1 chain, long enough for 10 chains. Show chains and poles.

15. Construct a scale of $\frac{1}{31000}$ to show Versts. 1 Verst = 1167 yds.

16. Draw a scale of miles and furlongs in which $1\frac{1}{2}$ furlongs equal $\frac{1}{8}$ of an inch.

17. A map is 40 inches long and 27 inches broad; it represents an area of 50 square miles. Draw the scale of the map to show miles, furlongs and diagonally chains.

18. A scale of 4 inches to the mile is attached to a map. The distance between 2 towns was found from the scale to be 19 miles and 4 furlongs while the real distance is 16 miles and 4 furlongs. The survey was known to be correct. Construct a correct scale to the map to read to miles and furlongs.

19. A horse passes over 260 yds per minute. Construct a scale of $\frac{1}{[illegible]}$ adapted to time.

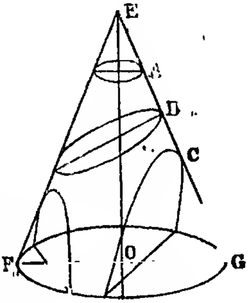
CHAPTER XII.

CONIC SECTIONS, ELLIPSE, PARABOLA AND HYPERBOLA.

Some of the curves used in engineering and mechanical drawings such as the ellipse, parbola, hyperbola, cycloids, involute, volute, spirals &c. cannot be drawn by the bow pencils or compasses. These curves are drawn by finding a number of points in them and then tracing the curve through these points freehand or with the help of curved pieces called French curves.

Some of these curves, ellipse, parbola and hyperbola are conic sections. A conic section is obtained by intersecting a cone by a plane.

The five different sections of cone are :—Fig. 193.



193.

1. A *triangle* is obtained when the cone is cut by a plane passing through its axis as EFG.

2. A *circle*, when the section plane passes at right angles to the axis EO of the cone as at A.

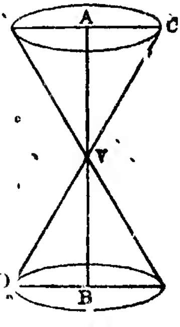
3. An *ellipse*, when the section plane cuts the cone obliquely without intersecting the base i. e. when the inclination of the section plane with the horizontal plane is less than the angle which the slant side makes with the base of the cone as at B.

Like the circular section it is a completely bounded curve.

4 A *parabola*, when the sectional plane is equally inclined with the slant side of the cone or is parallel to the generator. It cuts one side of the cone and the base. It is an open curve bounded on one side as at C.

5. A *hyperbola*, when the sectional plane is inclined at a greater angle than the angle at the base of the cone or eventually is parallel to the axis of the cone. The hyperbola cuts the two equal and opposite cones and consequently it has two branches which extend in opposite directions. Like parabola it is an open curve.

The double cone is formed thus :—Let AB and CD, two straight lines, intersect at V. Let CD revolve round AB which stands vertical, the inclination to each other and the point, of intersection V remaining fixed. The two equal and opposite right circular cones will be generated. AB is called the axis, CD the generator and V the vertex Fig. 194.

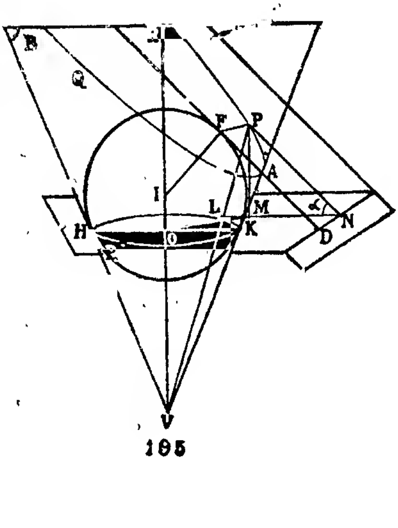


194.

The construction of these curves will be better understood if we regard them as being traced by a point moving on a plane surface according to some fixed law as in the circle the moving point always keeps the same distance from a fixed point, the centre of the circle.

A conic section is the curve described by a point which moves in a plane in such a manner that its distance from a fixed point in the plane, which is the focus of the curve, is in a constant ratio to its distance from a fixed line in the plane, called the directrix. Fig. 195.

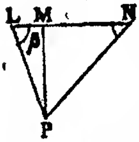
Let V be the vertex and VR the axis of a hollow cone. DNP represents a section plane cutting the cone in PAQ. Let a sphere centre I, be inscribed in the cone so as to touch the plane at F. This sphere touches the cone in a circle, the plane of which is perpendicular to the axis. Let this plane intersect the section plane in DN.



195

Select any point P on the curve PAQ. Drop PM perpendicular to the plane HK meeting it in M. Draw PN perpendicular to DN. Join PV cutting the horizontal circle HK in L. Join PF, ML and MN. The triangles LMP and NMP would appear if drawn on one plane as LNP (Fig. 196.)

PM is prependicular to the plane HK therefore it is prependicular to ML and MN drawn in the plane and meeting PM in M. The line PL is on the surface of the cone which ends at V, the vertex therefore it is a slant side of the cone and PM is parallel to VR the axis therefore the angle LPM is equal to half the vertical angle of the cone. Therefore the angle PLM is equal to the complement of half the vertical angle of the cone and is therefore constant for any position of P, hence the ratio PL : PM is constant.



196

Again PN : PM is constant as the angle PNM is constant i. e. the angle between the sectional plane and the horizontal plane HK.

Therefore the ratio PL : PN is constant.

But PL = PF tangents to the same sphere from a point P hence the ratio PF : PN is constant i. e the distance from the focus : the distance from the directrix is constant.

This ratio $\frac{PF}{PN}$ is called the eccentricity of the conic section.

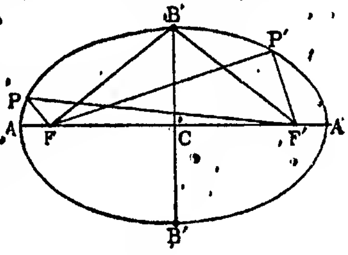
The point F is called a focus and the line DN is a directrix.

When PL or PF (Fig. 195) is less than PN i. e $\angle \alpha$ or $\angle PNM$ is less than $\angle \beta$ or $\angle PLM$ *i. e* when the angle of the section plane with the horizontal is less than the angle which the slant side of the cone makes with the horizontal, the section is an ellipse. When PL = PN *i. e* $\angle \alpha = \angle \beta$ the section is a parabola and when PL is greater than PN *i. e* angle α is greater than $\angle \beta$ the section is a hyperbola.

The ellipse and its properties :—

The ellipse has two foci and two directrices. The longest diameter of the ellipse is called the transverse diameter or the

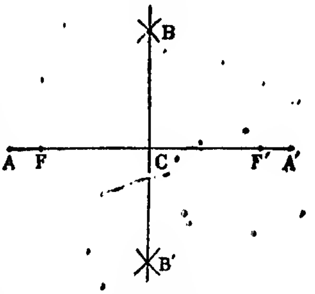
major axis, the shortest diameter is called the conjugate diameter or the minor axis. The two diameters bisect each other at right angles which is called the centre. (C in fig. 179)



197.

In any ellipse the sum of the focal distances from a point in the curve is constant and equal to the major axis (AA' in fig. 197).

Given A A' the major axis and BB' the minor axis of an ellipse to find out the two foci. Place the two axis bisecting each other at C. With B or B' as centre and C A or C A' *i. e.*, half the major axis as radius describe arcs cutting the major axis AA' in F and F' which are the required foci. Fig. 198.



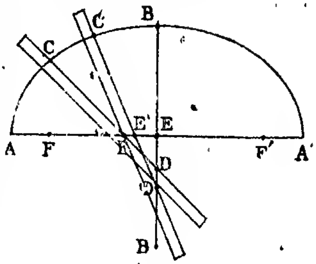
198

Given the major axis and the foci of an ellipse to determine the minor axis. Let A A' be the major axis and F and F' be the two foci. Bisect AA' in C. With F and F' as centres and radius CA or half the major axis describe arcs cutting each other in B and B'. Join BB' which is the minor axis. Fig. 198

137. Given the principal axis of an ellipse to construct the ellipse mechanically. Fig. 199.

(1) Place the two axis A A' and B B' bisecting each other at right angles. Take a slip of paper with one edge straight and set off on this edge the distance CD equal to half the major axis and the distance CE equal to half the minor axis. Place the strip in successive positions with the points E and D on the major and minor axis respectively and mark the corresponding position of C which will be points on the curve of the ellipse.

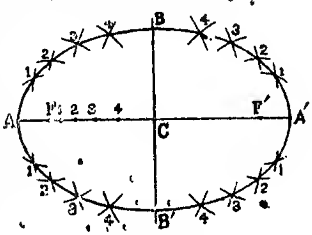
(2) Find F and F′ the two foci of the ellipse (Fig. 197). Fix two pins on the two foci and take a piece of thread little longer than AA′. Tie one end of the string on the pin at F and the other end on pin at F′ in such a way that the loose portion shall exactly be equal to AA′. By placing the point of a pencil inside the thread at P or P′ and keeping it drawn tight, the pencil on being moved would trace the ellipse.



199.

138. To draw an ellipse, the diameters being given. By means of intersecting arcs. Fig. 200.

Find F and F′ the two foci. C, the centre of the ellipse where the two diameters intersect. Take points between one of the foci and C, first close to the focus then further apart as 1, 2, 3 and 4 between F and C. From F and F′ as centres and with radii A1 and A′1, A2 and A′2, A3 and A′3, A4 and A′4 draw arcs intersecting on each side of A A′. Through these points draw a curve free hand which is the curve of the ellipse.

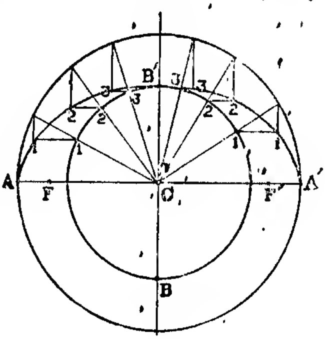


200.

139. The same by means of intersecting ordinates. Fig. 201

With C as centre and half the major axis as radius draw a circle and from the same centre and with half the minor axis as radius draw another circle inside the first. Take points in the circumference of the smaller circle as 1, 2, 3 &c. Join C1, C2 C3

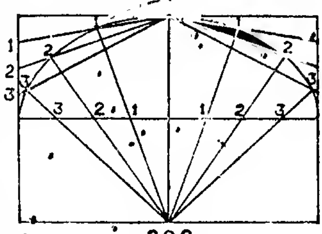
and produce them to meet the outer circumference in 1′, 2′, 3′ &c. From the points in the smaller circle draw lines parallel to AA′ the major axis and from the points in the outer circle draw lines parallel to BB′, the minor axis to meet the corresponding lines first drawn parallel to AA′. The points of intersection are points in the curve of the ellipse which can be drawn by joining them free hand.



201

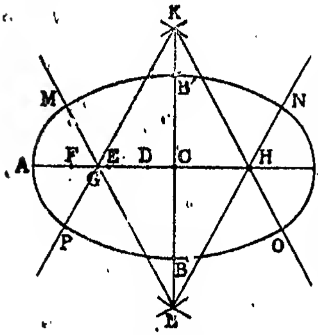
140. The same by means of intersecting lines. Fig. 202.

Draw the lines AA′ and BB′, bisecting each other at right angles at C. Draw DE and FG parallel to AA′ through B and B′ and draw DF and EG parallel to BB′ through A and A′. Divide AD and AC into the same number of equal parts say four. Similarly divide A′C and A′E each into 4 equal parts. Number these divisions from A and A′ each way similarly. Join B with the points in AD and A′E and join B′ with the points in AC and A′C and produce them to meet the corresponding lines drawn from B i.e. B′1 meets B1, B′2 meet B2 &c. The points thus obtained are on the curve of ellipse which will be drawn half by joining them. The lower curve can similarly be drawn by dividing AF and A′G.



202.

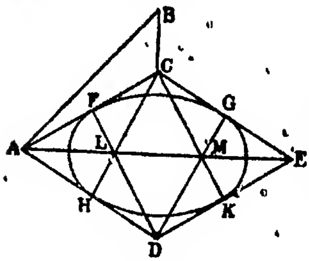
141. To construct an elliptic figure by means of arcs of circles when the major and minor axes are given. Fig. 203.



203

Let AA' and BB' bisect each other at right angles at C. From one end o AA' take a distance equa to the minor axis as A'D Divide DA the remaining portion of the major axis into 3 equal parts in E and F. Take 2 of these parts, DF as radius and from C as centre draw arcs cutting A'A in G and H. With G and H as centres and with GH as radius draw arcs intersecting each other in K and L. Join LG, LH, KG, and KH and produce them. From K and L as centres and with KB and LB' respectively as radii draw arcs to meet KG and KH, LG and LH produced in P and O and M and N respectively. From G and H as centres and with radius GA or HA' draw arcs completing the ellipse.

142. The same, when only the major axis is given Fig. 204.



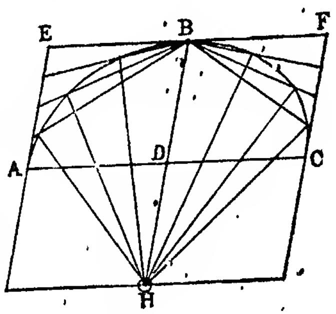
204.

Let AB be the major axis. Draw AC at an angle of 15° and BC at 45° with AB from the two ends of AB and meeting in C. Draw AD at an angle of 60° with AC and equal to it. From C and D draw lines parallel to AD and AC respectively to complete the rhombus ADEC. Bisect AC, AD, EC ED in F, H, G and K respectively. Join DF, DG and CH, CK intersecting in L and M. With D as centre and DF as radius draw an arc FG and with C as centre and CH as radius draw an

arc HK. Complete the ends with L and M as centres and LF and MG as radii.

143. To draw an ellipse to pass through three given points A, B and C by joining which a triangle may be formed. Fig. 205.

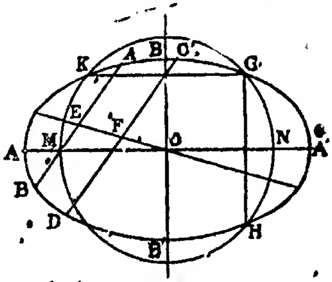
Let ABC be the three given points. Join AC. Bisect AC in D. Join BD and produce it to H making DH=BD. Through A and C draw AE and CF parallel to BD and through B draw EBF parallel to AC. Divide AD, AE and CD, CF into any number of equal parts say 4. Proceed like problem 140 to obtain points on the curve of the ellipse.



205

144. To find the centre, axes and foci of a given ellipse. Fig. 206.

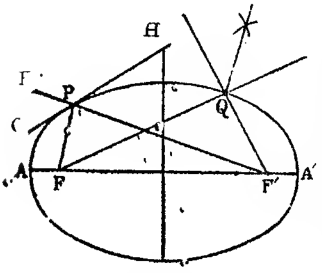
Draw any two parallel chords in the ellipse as AB and CD. Bisect them in E and F. Join EF and produce it both ways to meet the ellipse. The line passing through E and F is a diameter *i.e.* it passes through the centre of the ellipse. Bisect it at O. Then O is the centre. With O as centre and with a convenient radius draw



206

an arc cutting the ellipse in K, G and H. Join KG and GH. Draw lines parallel to KG and GH through O which are the axes of the ellipse. The foci can be found by intersecting the major axis with arcs from one end of the minor axis as centre and half the major axis as radius.

145. To draw a tangent and a normal to an ellipse from given points in the curve. Fig. 207.



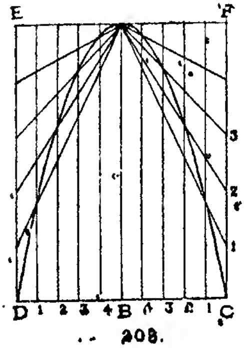
207.

Let P and Q be two points on the ellipse. Find the axis and the foci F and F′. Join the foci with P and Q. Produce F′P to E and bisect the angle EPF by the line GPH which is the tangent to the curve at P. The normal at P is the line which is perpendicular to the tangent GPH at P. The normal can also be found thus:—Produce FQ and F′Q, the line which bisects the outer angle thus formed is normal to the curve at Q.

The Parabola.

146. To construct a parabola, an abscissa and a base or double ordinate being given. Fig. 208.

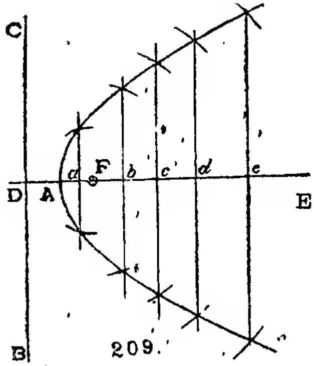
Let AB be the abscissa and CBD be the base or double ordinate. Complete the rectangle EFCD. Divide BC the ordinate into any number of equal parts say 5 parts. Divide CF into the same number of equal parts and join A, the vertex with the divisions in CF. Number the points from C both ways along CF and CB. From the divisions in CB draw lines parallel to AB and where these lines intersect the corresponding lines from A to the points in CF are the points in the curve of parabola. Repeat the operation on the other side to complete the curve.



208.

147. To find points for drawing a parabola the focus F and the directrix CB being given. Fig. 209.

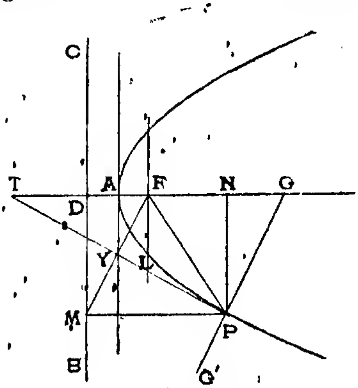
Draw the line EFD through F perpendicular to CB, the directrix which will be the axis. Bisect FD in A which will be the vertex, of the curve. Take any points a, b, c, d, e in the axis and draw perpendiculars through them. From F as centre, mark off on the perpendiculars, respectively up and down, with radii equal to aD, bD, cD, dD, eD. The points thus formed are points on the parabola. Join the points and complete the curve through A.



209.

148. To draw a tangent and normal to a parabola at a given point P. Fig. 210.

Join FP. From P draw PN perpendicular to the axis. Set off FT on the axis produced and make AT=AN or FT=FP. Join TP then TP is the tangent at P. Or from P draw PM perpendicular to CB. Bisect the angle FPM by PT which is the tangent. Draw PG perpendicular to PT the tangent then, PG is the normal.



210

Properties of the parabola :—

The following properties of parabola, are given here for the convenience of students for solutions of problems. Fig. 210.

1. The tangent PT bisects the angle FPM.

2. A line from the focus perpendicular to a tangent meets the latter at a point where the tangent at the vertex meets it.

3. Draw PN perpendicular to the axis then AT = AN.

4. Through the focus draw the double ordinate LFL which is called the latus rectum and it is equal to 4 AF.

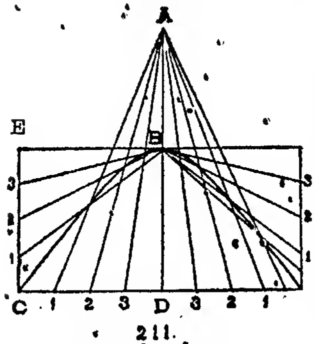
5. PG is drawn perpendicular to TP i. e. it is the normal at P. The length of the subnormal NG = $\frac{1}{2}$ LL′ = 2 AF = FD.

6. The area of the figure ALPNA is two thirds the circumscribing rectangle AYXPN.

The hyperbola.

149. To construct an hyperbola, the diameter, an abscissa and an ordinate being given Fig. 211.

Let AB be ~~the~~ diameter, CD an ordinate and BD, abscissa.

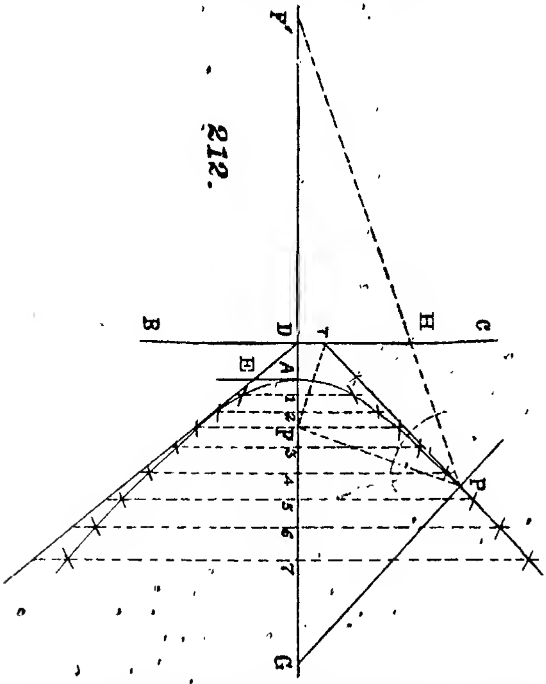


211.

Through B draw a line parallel to CD and complete the rectangles on both sides of BD. Place AB and BD in the same straight line i.e. produce DB to A and make the produced portion equal to AB.

Divide CD and CE into the same number of equal parts say 4. Number the points each way from C. The divisions on CD the ordinate is to be joined to A and those on CE, to B. The intersections of corresponding lines give the points on the hyperbola.

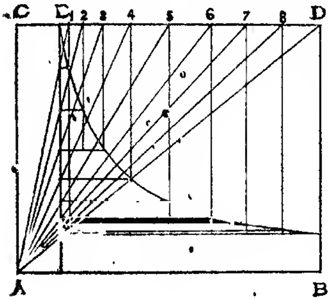
150. To describe the curve of a hyperbola, the focus directrix and vertex being given. Fig. 212.



Let F be the focus, CD the directrix and A the vertex. Join FA and produce it both ways. At A draw AE perpendicular to AF and make AE = AF. D is the point where FA produced meets the directrix. Join DE and produce it. Take any number of points on the axis as 1, 2, F, 3 and 4 and through these points draw lines perpendicular to the axis both up and down meeting DE produced in 1, 2, f, 3 and 4. From centre F with radii 11, 22, Ff, 33, 44 intersect the double ordinates through 1, 2, f, 3 and 4 respectively. The points thus obtained are on the hyperbola. ∴ AF > AD.

151. To draw a rectangular hyperbola as used by engineers. Fig. 213.

Let AB and AC represent two axis and E the vertex of the curve. Complete the rectangle ABDC. Take any number of points between E and D as 1, 2, 3, 4, 5, 6, 7 and 8. Draw a line from E parallel to CA. Join the points 1, 2, 3, 4, 5, 6, 7, 8 and D with A intersecting the line from E in 1, 2, 3, 4, 5, 6, 7, 8, 9. Through the two sets of points in ED and E9 draw lines parallel to AB and AC. They intersect in points which joined will give the rectangular hyperbola.



213.

152. To draw a tangent and a normal to the curve of hyperbola. Fig. 212

Let P be a point on the hyperbola. Join PF. Draw FT perpendicular to FP meeting the directrix in T. Join TP then TP is a tangent. Draw PG perpendicular to TP at P then PG is the normal.

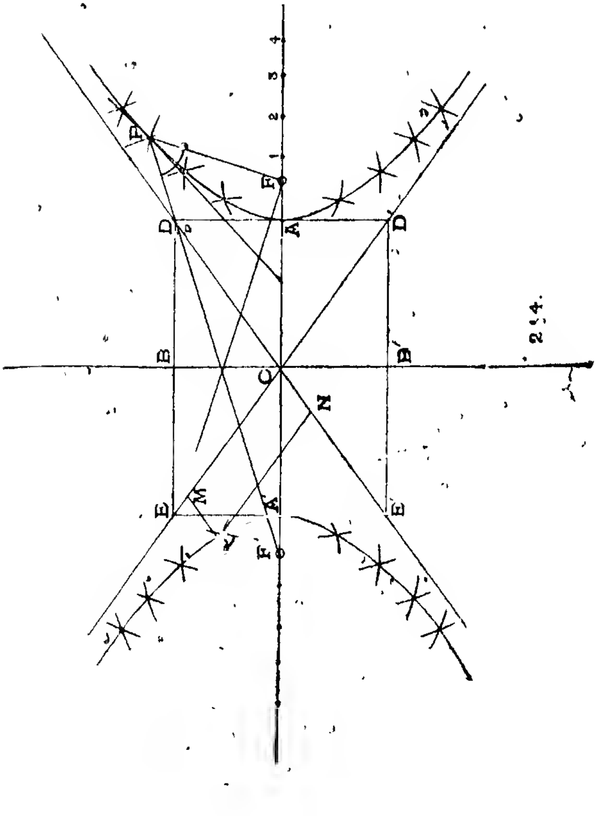
Make the angle TPH equal to the angle FPT The line PH produced meets the axis produced in a point which is the focus of the other branch of the curve. AF>AD

153. Properties of hyperbola. Fig. 214.

Hyperbola has many closely allied relations to ellipse. Like the ellipse hyperbola has two axes, two foci, two directrices and a centre.

1. Let AA′ be the major axis. Bisect it at C, then C is the centre. Through C draw BCB′ perpendicular to AA′. BCB′ is the minor axis in length less than AA′ the major axis.

2. Draw lines parallel to AA′ and BB′ through A, A′ B, and B′. Join the dioganals ED and EO of the rectangle thus



214.

formed. Produce the two diagonals both ways. They are called the asymptotes to the curve. Asymptotes are lines which, as they recede to infinity with a curve, approach nearer and nearer to the latter without limit, but never actually coincide with it.

3. In the ellipse the sum of the focal radii is equal to the transverse or major axis, in the hyperbola the difference of the focal radii is equal to the major axis.

4. The distances of the foci from the centre C is equal to the diagonals CD or CE

5. In the ellipse the normal bisects the angle between the two focal radii, in the hyperbola the tangent bisects the focal radii.

6. Take any point Q on the hyperbola, draw lines parallel to the asymptotes from it meeting them in M and N. Then QM × QN is constant, an important property of hyperbola.

EXERCISE ON CHAPTER XII.

1. The foci of an ellipse are $2\frac{1}{4}''$ apart and the major axis is $4\frac{1}{4}''$ long. Draw the ellipse and draw a tangent from any point on the curve.

2. Draw a parabola by intersecting lines, axis, $1\frac{1}{2}''$, double ordinate, $2\frac{1}{2}''$.

3. With a diameter is 4", an ordinate 1·8" and an abscissa 1·4", construct a hyperbola.

4. Draw a rectangle 2·75" × 2" and inscribe an ellipse within it.

5. Draw a rectangle 3" × 2" and let two adjacent sides represent the axes of a rectangular hyperbola. Measure 1/2 an inch from one corner on one of its longer edges and let this point represent the vertex of the curve. Complete the hyperbola.

6. The transverse axis of a hyperbola is $2\frac{1}{4}''$ and the distance between the foci is $2\frac{3}{4}''$. Determine the conjugate axis, the asymptotes and draw a portion of the curve.

7. Draw any parallelogram, and in it inscribe a parabola which touches one side as its middle point, and passes through the ends of the opposite side. Determine the latus rectum.

8. Draw the tangent and the normal at a point on an ellipse and on a parabola.

9. Draw an ellipse to pass through three given points.

10. Draw an ellipse given one axis and a point on the curve of the ellipse.

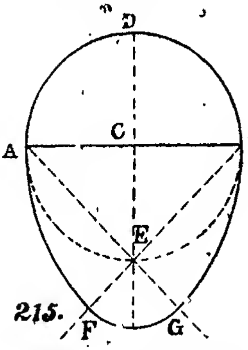
11. Given the two foci of an ellipse and the sum of the distances of a point in the curve from the two foci. Construct the ellipse.

CHAPTER XIII.

PLAIN CURVES OTHER THAN CONIC SECTIONS.

153 To construct an oval or egg shaped figure the width being given. Fig. 215.

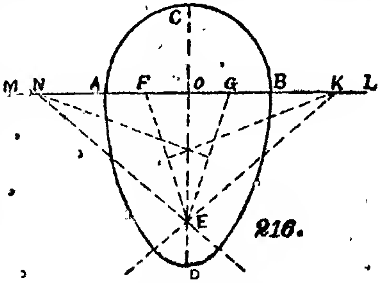
Let AB be the given width. Bisect AB at C. Draw a circle on AB as diameter. Though C draw DCE perpendicular to AB meeting the circle in D and E. Join AE and BE and produce them indefinitely. With centres A and B and radius AB describe arcs meeting AE and BE produced in G and F respectively. With E as centre and radius EF, or EG complete the figure.



215.

154. To construct an oval when the height and the width are given. Fig. 216.

1st. AB the width and CD the height are given. Bisect AB at O. Place CD at right angles to AB through O; the portion OC = OA or OB. With O as centre and OA as radius draw a semicircle. D is the lowest point of the egg shaped figure.



216.

Bisect AO at F and OB at G. Take DE = $\frac{1}{4}$ AB. Join FE and GE. Bisect FE and GE at H and I. Draw HK and IN perpendiculars to FE and GE meeting AB produced in N and K

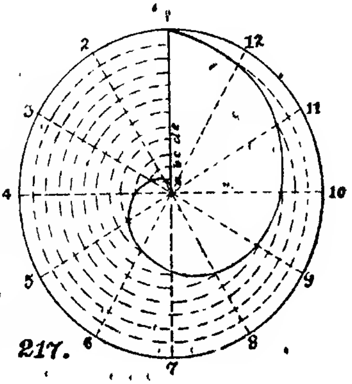
With N and K as centres and radius NB or KA draw arcs till they meet the lines KE and NE produced. Complete the curves with E as centre and ED as radius.

2nd. Take AM and BL each equal to AO on both sides of AB produced Join ME and LE. With M and L as centres and MB and LA as radii draw arcs till they meet ME and LE produced. Complete the curve with E as centre and ED as radius.

155. To construct a spiral of any number of revolutions. Fig. 217.

1st Archimedean spiral

Draw a circle and divide it by radii into a number of equal parts say 12 as 1, 2, 3, 4, 5, 6 &c Divide the radius No. 1 into 12 equal parts and number them as *a*, *b*, *c*, etc. Let *o* be the centre of the circle. With centre *o* and radius, oa mark on radius No. 2, a'. With radius ob mark on radius No. 3, b'. With radius oc mark on radius No 4, c' and so on till the 12 divisions are finished. Join the points *o*, a', *b'*, *c'*, d', &c. thus found by a fair curve which is the spiral.



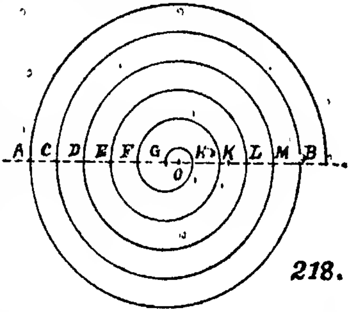
217.

156. To draw a common spiral of five revolutions on a given diameter AB by means of semicircles Fig. 218.

Divide AB into 10 equal parts. Bisect one of the two middle divisions say the 6th from A in O. Let the divisions be named from the left as AC, CD, DE, EF, FG, GH, HK, KL, LM, MB.

Draw a semicircle on GH the sixth division. Then draw a semicircle on FH (2 divisions) on the opposite side of the 1st semicircle. Then draw a semicircle on FK on the same side

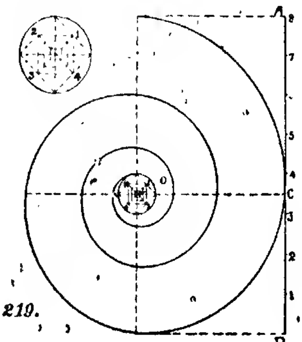
as the 1st semicircle, then draw a semicircle on EK on the other side, then a semicircle on EL, then on LD, then on DM, then on MC, then on CB, then on AB to continue the spiral. It will be seen that the centre for the upper semicircles is O and for the lower semicircles, the point G. The upper semicircles are on odd number of parts and the lower ones are on the even number of parts.



218.

157. To draw a spiral adopted to the volute of an Ionic Column. Fig. 219.

Let the height of the volute be given as AB. Divide the given height into 8 equal parts. Bisect the 4th part in the point C and from it draw a line perpendicular to AB and towards the left of it for the right volute. Make this line equal in length to four of the divisions on AB which will give O the eye of the volute. With O as centre draw a circle with radius equal to C4. Inscribe a square in this circle and bisect each of its sides in 1, 2, 3 and 4. Join these points and draw diagonals. Divide each semi-diagonal into 3 equal parts and join them, thus making two more squares inside the other. The corner of each of these squares in succession is the centre of each of the quadrants commencing from the left upper corner of the outer inscribed square which is numbered as 1, then proceed right hand. Finish



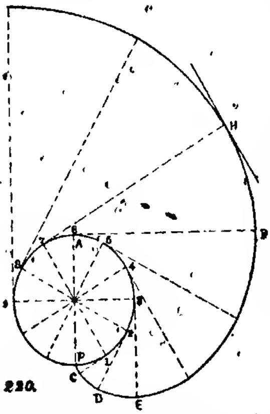
219.

the four corners of the outer square, then commence from the left upper corner of the 2nd square and so on. The curve will turn from the right to the left for the right volute and from the left to the right for the left volute in which case the line CO is to be drawn to the right of AB. The eye is drawn enlargd in the figure in the corner, points numbered for the left volute.

158. To draw the involute of a circle. Also to draw a tangent to the curve at any given point. Fig. 220.

If a perfectly flexible thread be unwound from the circumference of a circle and kept constantly stretched, the extremity of the thread describes a curve known as the involute of a circle.

Let AP be the circle, and P the generating point. Draw the diameter AP. At A draw the tangent AB. Make AB=semi-circumference of the circle (prob. 116). Divide AB and the semi-circumference into the same number of equal parts, say 6. Number the divisions of the semi-circle as 1, 2, 3, 4, &c. and also number the divisions in AB similarly. Draw tangents to the circle at points 1, 2, 3, 4 &c. Make 1C=A1, 2D=A2, 3E=A3 and so on. To obtain points beyond B proceed in the same manner and take lengths on the tangents =n times a divison on AB according to the number of the tangent.



220

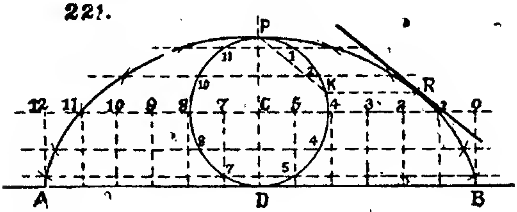
To draw a tangent from a point in the involute curve:—Draw a line from the point tangential to the circle AP, the tangent is perpendicular to this line.

The curves of *Cycloid, Epicycloid* and *Hypocycloid.* The curve described by a point on the circumference of a circle which rolls (1) on a straight line in a plane is called the Cycloid, (2) when it rolls externally on the circumference of another circle the curve is called Epicycloid and (3) when it rolls internally on the circumference of another circle it is called Hypocycloid.

The moving circle is called the *generating circle.* The line or circle on which the generating circle rolls is called the *director* or *base* and the point on the circle tracing the curve is called the *generator.*

159. To draw the curve of Cycloid. Fig. 221.

221.

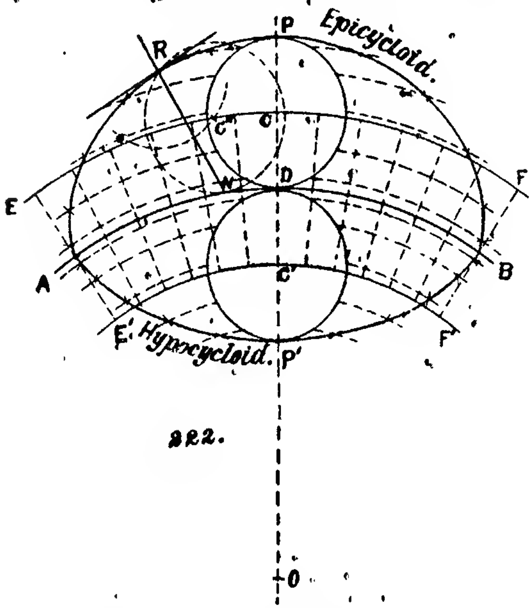


Let AB be the director. PD, the generating circle, the point P, the generator. Let D be the point where the generating circle touches the director AB. Draw the diameter DP and bisect it at C. Through C the centre of the circle draw a line parallel to AB. Make AD and DB each equal to half the circumference of the generating circle either equal to $3\frac{1}{7}$ CD or by problem 116. Divide the circle into 12 equal parts and divide the line ADB also into 12 equal parts. Draw lines from the points in AB perpendicular to it to meet the line through C dividing it into 12 equal parts. Through the points in the circumference draw lines parallel to AB. Number the points from D to P and similarly from C to the right on the line passing through it, say 5, 4, 3, 2 and 1 in both the generating circle and the line. With centre 5 in the middle line and radius equal to CP intersect the line through point 1 in the circumference. Similarly with point 4 as centre intersect the line through 2 and so on. Joining these points the right half of the cycloid is obtained when the generating circle rolls from D to B. The left half is similarly drawn.

160. To draw a tangent to the curve of Cycloid Fig. 221.

Take any point R in the curve. Draw RK parallel to AB meeting the circumference of the generating circle in K. Join PK. Through R draw a line parallel to PK which will be the tangent.

161. To draw the curves of the Epicycloid and the Hypocycloid. Fig. 222.



222.

Let AB be the director which is a part of a circle. Let PD be the external generating circle with point P as the generator and P'D be the internal generating circle with point P' as the generator. The external point P will trace the curve *epicycloid* and the internal point P' will trace the curve *hypocycloid*. Let C and C' be the centres of the two generating circles. Take lengths from D on the director both ways equal to the semi-circumference of the generating circle as DA and DB Divide the circumference of the generating circle and the director AB into 12 equal parts commencing from D, half the number on the right and half on the left.

Draw arcs of circles through C and C' from O, the centre of

the director and join the points of division in the director with the centre and produce; these lines will give points of division on the arcs of circles through C and C′. From the points of division on the generating circles draw arcs parallel to the director. Intersect these arcs by arcs drawn from the points in the circles through C and C′ with the radius of the generating circle. The upper points are the points for the epicycloid and the lower points, for the hypocycloid.

162. To determine the tangent and the normal at any point R on the epicycloid or the hypocycloid.

Take any point R on the curve; with R as centre and with the radius of the generating circle intersect the arc passing through C at C″. This is the position of the rolling circle when the tracing point is at R. With C″ as centre and with radius CP draw a circle passing through R. Find the point of contact of this circle with the director which is N. Join NR which is the normal to the curve. Draw RT perpendicular to NR at R which is the tangent. The tangent and the normal is shown on the epicycloid. The construction is the same for both the epicycloid and the hypocycloid.

CHAPTER XIV.

ARCHES.

The curves of arches used for Engeneering works are all arcs of circles either drawn from one centre or from more than one centre.

Explanation of terms. Fig. 223.

The clear distance from wall to wall on which the arch stands is called the *span* of the arch as AB. Fig. 223.

The points from which the arch springs is termed the *springing points* and the line which joins the side of the wall with the lower face of the arch is the *springing line.*

The walls on which the arch stands or rests are the *abutments.*

The line drawn from the middle of AB the span and perpendicular to it to meet the lower face of the arch (as DC in the figure) is the *height* or the *rise* of the arch.

The point where the height of the arch meets the lower face of the arch as C in the fig. is calld the *crown* of the arch.

The inner face ACB is the *intrados* or *soffit* of the arch. It is the concave surface of the arch.

The outer face FEG is the *extrados* or *back* of the arch, the convex surface of it.

A portion of the arch near the springing is called the *haunch* of the arch. It is nearly a third of the arch from each springing.

The stone at the crown of the arch is the key stone.

The triangular open space between the back of the arch and the horizontal tangential line through E the top of the arch is called the *spandril space, sl* in the fig.

The span and rise of all arches are given to draw the arches:—

163. To construct a semi-circular arch when it is tilted 6″. Fig. 224.

Let AB be the span. Draw CD a line parallel to AB and 6 inches above it. Through A and B draw lines perpendicular to AB meeting CD in C and D. Bisect CD and draw a semi-circle on it. Then the arch ACEDB is the tilted semi-circular arch, often used in verandah openings when mouldings project at A and B.

164. To construct a segmental arch. Fig. 225.

Let AB be the span and DC the rise. Place DC at right angles to AB at its middle point. Join AC and BC. Bisect AC and BC at E and F and from E and F draw EO and FO perpendiculars to AC and CB meeting at O which is the centre of the circle. With O as centre and OA as radius draw the soffit of the arch ACB. From A and B draw radial lines AG and BH and make them equal to the thickness of the arch. AG and BH are the springing lines of the arch. With O as centre and OG as radius draw the extrados of the arch.

165. To construct an equilateral Gothic arch. Fig. 226. Here only the span AB is given.

Bisect AB at D and draw DC perpendicular to AB. With A as centre and AB as radius draw the arc BC meeting the line DC in C. With B as centre and BA as radius draw the arc AC. It is the simplest form of Gothic arch.

166. To draw the lancet arch when only the span is given. When the height of the Gothic arch is more than the span or equal to it, it is called the lancet. Fig. 227.

Let AB be the span. Bisect AB at D; and produce AB both ways. With A and B as centres and half of AB as radius draw semicircles cutting AB produced in E and F. With E and F as centres and radii EB and FA draw arcs meeting each other at C.

167. To draw the lancet arch when both the span and the rise are given. Fig. 228.

Let AB be the span and DC the rise. Place DC perpendicular to AB from its middle point. Join AC and CB. Bisect AC and CB at G and H and from G and H draw lines perpendiculars to AC and BC to meet AB produced in F and E. Then F and E are the centres and FA and EB are the radii respectively.

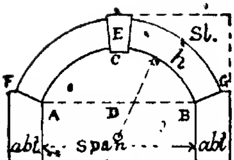
168. To construct a four centered Gothic arch. Fig. 229. Here only the span is given.

1st method. Let AB be the span. Divide AB into 4 equal parts in E,D and F. At A and B draw AG and BH perpendiculars to AB

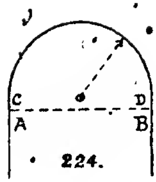
and make them equal to it. Join G with F the 2nd point from B and produce and join H with E, the 2nd point from A and produce. E. F. H and G are the four centres. With E and F as centres and with EA and FB as radii draw arcs till they meet the lines HE and GF produced in K. and L. With G and H as centres and radii equal to GL and HK finish the remaining portion of the curve.

2nd method. If less height is required for the arch divide the span into 6 or 8 or more equal parts and draw perpendiculars from the 2nd points from the two ends and make them equal to the span. Join the feet of the perpendiculars with their tops in the opposite directions, and produce and draw the arch like the preceeding one. Fig. 230.

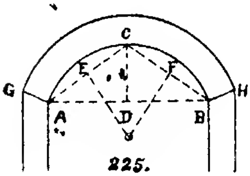
3rd method. The four centres may be obtained thus. On the middle two divisions draw an equilateral triangle downwards and produce the sides downwards and obtain another equal equilateral triangle with its base downwards. The two corners of the base of this lower triangle and the two 2nd points from the two ends of the span are the 4 points. Fig. 231.



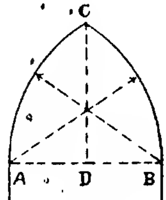
223.



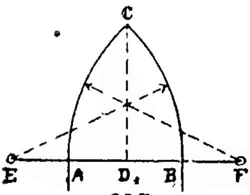
224.



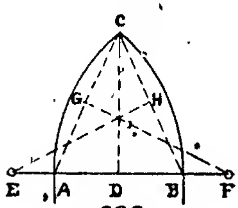
225.



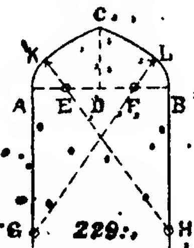
226.



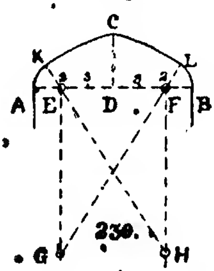
227.



228.



229.



230.

169. To construct a semi elliptic arch. Fig. 232.

The span and the height are given.

Let AB be the span and DC the rise. Join AC. Take CE equal to the difference of the two semi axes *i.e.* AD-DC and bisect the remainder EA at F. From F draw FG perpendicular to AC meeting AB in G and produce it to meet CD produced in O. Take DH = DG. Join OH and produce. Then G, H and O are the three centres. With G as centre and GA as radius draw the arc AK meeting OG produced in K and with O as centre and OK as radius draw the arc KC. Finish the remaining portion similarly.

170. To construct a horse shoe or Moorish arch. Fig 233.

When the arc of the arch is more than a semi-circle it is called a horse shoe or Moorish arch. AB is the span DC is the rise. Take O in DC so that OC will be more than AD the half span. With O as centre and OC as radius draw the arc ACB.

171. To construct an Ogee arch when the span and the height is given. Fig. 234.

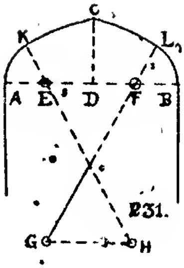
The height of the Ogee arch is always more than half the span.

Let AB be the span and DC the height. Join CA and CB Bisect CA and CB in E and F. Bisect AE aud BF in G and H. Draw GK and HL perpendiculars to AE aud BF meeting AB in K and L. Through C draw a line parallel to AB. Bisect EC and FC in P and M. Draw PQ and MN perpendiculars, then Q and N are the two centres for the upper portion and K and L, two centres for the lower portion of the arch.

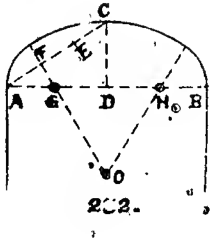
172. To construct the pointed trefoil arch. Figs. 235 and 236.

Let AB be the span and DC the rise which should be nearly equal to the span, may be little less or more. Join AC, BC. Divide AC and BC each to 4 equal parts. Join the middle parts E and F.

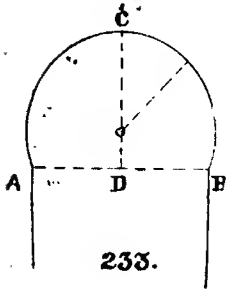
The construction is plain from the figure.



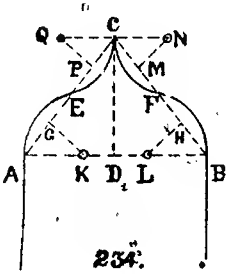
231.



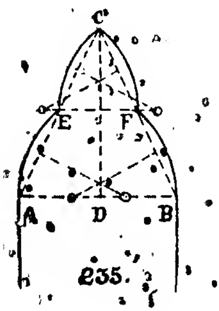
232.



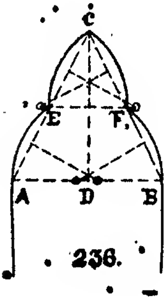
233.



234.



235.



236.

Drawing Test Examination Questions for admission to the Engineer Class.

(Five questions are usually required to be answered in three hours and drawing, to be executed in bold pencil lines.)

1923.—Print the word "Builders" in plain block $\frac{1}{2}$ inch high; and write the following sentence in italics:—"By means of a sextant the Surveyor found the angle DAB to be 25°.

1923.—Draw a triangle in which the sides are in the ratio 3 : 4 : 6, the longest side to be 3 inches long.

1923.—Draw an octagon each side of which measures one inch. Enlarge the figure so that one side of the octagon on the enlarged drawing may represent three feet to a scale of 1 inch = :ft

1923.—An arch for a bridge spanning a river 90 feet wide is to be a semi-ellipse. The crown of the arch is to be 25 feet above the normal level of the river(i.e half minor axis=25 feet). Draw the outline of the arch to a scale of 20 feet to the inch. (Span = major axis). Also draw normals to the outline at intervals of 20 feet.

1922.—Print the word "Section" in plain block $\frac{1}{2}$ " high.

Write the following in italics:—

"The art of any craft, such as carpentry, can only be learnt in one way, by actually handling the tools. No amount of study will ever help a man to make a mortise and tenon joint. This can only be learnt by doing."

1922.—Determine $\frac{1}{96}$ of 2 inches by means of a diagonal scale.

1922.—A piece of wire 12″ long is bent at two points in such a way as to make a triangular template, the angle at the vertex of the triangle being 52° and its altitude is 3.4″. Construct the triangle and write down the lengths of its sides and the magnitude of its angles.

1922.—Two pulleys, $1\frac{1}{2}$ feet and 3 feet diameter respectively, are fixed at 6 feet centre to centre. An endless belt passes over the pulleys. Draw the elevation of the pulleys and belt to a

scale of 1 foot = 1 inch. Find the length of the belt. All construction lines must be shown.

1922.—Draw a pentagon of 1 inch side and enlarge it so that the area may be twice that of the one-inch pentagon.

1921.—Construct a scale of 9 feet to the inch. It should be long enough to measure 50 feet. Each main division to read 5 feet. The first division on the left of the scale to be subdivided to read to 6 inches.

1921.—Two roads meet at an angle of 45°, and at the point of meeting two cyclists, A and B, start along each road, A travelling at 8 miles, and B at 5 miles an hour. When B has travelled $7\frac{1}{2}$ miles, how far will A be from the starting point? Find the result by a geometrica construction.

1921.—Describe a circle of $1\frac{1}{2}$ inches radius. Draw any diameter A B. From a point in AB produced, draw a tangent 2 inches long to the circle.

1921.—Print the word "Engineer" in block $\frac{1}{2}$ inch high.

Write the following in italics:— "When using the T-square, hold and move it with the left hand, and draw the lines from left to right."

1921.—Draw any irregular quadrilateral figure, then draw a line from one corner which will divide the quadrilateral into two equal parts.

1919.—ABCD is a trapezium, the angle at A is 120°, AB is 2″ AD is 1″, BC = 2″ and DC = $\frac{3}{4}$″. Bisect the trapezium by a straight line from A.

1919.—A landowner had a square plot of land, and wished to keep for his own use a square equal to a quarter of its area and to divide for the use of his four sons the remaining three quarters into four parts, the same size and shape. Draw a square of 3 side and show how this can be done.

1919.—Two points are $1\frac{1}{2}$″ a part, and their distances from a straight line are $\frac{5}{8}$″ and $1\frac{3}{8}$″. Draw a circle to pass through the two points and to touch the straight tine.

1919.—A distance of 11 miles and 3 furlongs is represented

on a map by $4\frac{1}{3}''$. Draw the scale of the map showing furlong diagonally. What is the R. F. of the scale?

1919.—The distance between the foci of an ellipse is $2\frac{1}{2}$ and the major axis is $3\frac{1}{4}''$ long. Draw the ellipse.

1918.—Construct an isosceles triangle with its equal side $2\frac{1}{2}''$ long and the included angle 30° On the same base describe another isosceles triangle with its vertical angle double that of the first triangle.

1918.—Construct a square of $2\frac{3}{4}''$ sides, and through one corner draw a line cutting off $\frac{1}{3}$rd of its area.

1918.—Draw an ellipse, the major and minor axes being $3'$ and $2''$ respectively.

1918.—Print the word "Surface" in plain block type $\frac{1}{2}''$ high Print in italics (capitals $\frac{3}{16}''$ and small letters $\frac{1}{8}''$ high) the whole question, "Construct a square of $2\frac{3}{4}''$ sides" &c.

1918.—A line 90 feet long is represented in a drawing by a line $6''$ long. Make a scale of feet for the drawing and give its representative fraction.

1918.—Construct a regular polygon on the chord of an arc of 72°.

1917.—Print the word "Universal" in plain block type, $\frac{1}{2}$ inch high, and the following in italics (capitals $\frac{3}{16}$ inch and small letters $\frac{1}{8}$ inch high). "Civil Enginering College, Sibpur, Howrah."

1917.—A triangle has its sides $1\frac{1}{2}''$, $2''$ and $2\frac{1}{2}''$ respectively. Construct the figure and draw a square equal to half the area of the triangle.

1917.—Describe a segment of a circle having a chord of 3 inches and containing an angle of 150°.

1917.—Construct a diagonal scale of 10 feet = 1 inch to read inches. Mark off a length of $23'$-$8''$ on the scale itself.

1917.—Within a circle of $1\frac{1}{2}$ inches radius inscribe a regular hexagon. Within the hexagon inscribe three equal circles touching each other and each touching the two sides of the hexagon.

1917.—Draw a parabola, given the axis $2\frac{1}{2}$ inches and the double ordinate 5 inches.

1916.—The length of a building is 42′·16″. This length is represented on the plan by 5″. Draw a suitable scale for this plan.

1916.—Print the word "Plan" in plain block type $\frac{1}{3}$″ high.

1916.—Write your name and address in italics. The capital letters to be $\frac{3}{20}$″ high.

1916.—Draw a line AB, 4″ long. Draw a line BC, 3″ long, perpendicular to AB. With centre C and radius $\frac{1}{2}$″ draw a circle. Join AC cutting the circumference in D. Draw another circle to touch AB, and the first circle at D.

1916.—Draw a rectangle ABCD, AB = 5″, BC = 3″. Mark a point E midway between A and D. From E draw a line making an angle of 30° with AD and cutting AB in F. Reduce the figure EFBCDE to a triangle having the same area.

1916.—Draw by means of semicircles a common spiral of 3 revolutions on a diameter of 5″.

1923.—Make a freehand sketch of any one of the following :—Padlock, Chair, Three legged stool, Bucket, "Kodali.' Your sketch should be 6 inches long and proportionately wide.

1921.—Make a freehand sketch, from memory, of any one of the following articles, so as to be intelligible to all men :—Hand-saw, chair, road-roller, mallet, bucket.